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Creative Heritage of Vyacheslav Sergeevich Tanaev: Seventieth Anniversary

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Abstract—The main stages of life and scientific activity of V.S. Tanaev, as well as his basic scientific findings in the scheduling theory, decomposition of the optimization problems, and other fields of the applied mathematics are outlined in brief.

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1. MAIN STAGES OF LIFE

An outstanding Belarussian scientist, member of the National Academy of Sciences Vyacheslav Sergeevich Tanaev (1940–2002) would be seventy on March 28, 2010. His heritage is represented by numerous scientific works and the internationally acknowledged school of the scheduling theory and decomposition of the optimization problems that he founded.

V.S. Tanaev was born in the village of Akulovo of the Teblesh region in the Tver province of the USSR. After finishing in 1957 the secondary school in Simferopol, he entered the department of physics and mathematics of the Crimean pedagogical institute where he published his first scientific paper "On Mechanical Generation of Planar Curves." During his last year at the institute V.S. Tanaev combines studies with teaching at an eight-year school. In 1962 he graduated cum laude from the institute as teacher of mathematics and drawing. In the same year he started post-graduate studies at the Institute of Heat- and Mass-exchange of the Byelorussian Academy of Sciences (AN BSSR) where he planned to study the optical radiation of the nuclear explosions. However, soon he got interested in the works on operations research carried under the guidance of Academician (then Corresponding Member) of AN BSSR D.A. Suprunenko. Vyacheslav Sergeevich took great interest in this area and, after changing the theme of his dissertation, started to work actively on the problems of discrete optimization under the guidance of Dr. A.Sh. Blokh. In two years V.S. Tanaev wrote the candidate dissertation on "Solution of Special Classes of the Scheduling Theory Problems" which was defended in June of 1965. At the beginning of 1965 V.S. Tanaev joined the Institute of Technical Cybernetics (ITC) of AN BSSR which was just established on the basis of the cybernetic laboratories of the Institute of Mathematics and Computer Engineering. Since then, actually the entire scientific and labor activity of Vyacheslav Sergeevich was related with ITC.

In 1966 V.S. Tanaev organized a discrete-programming laboratory oriented to the problems of the scheduling theory and development of methods for solution of complex optimization problems in design and control. In 1970 he joined the Minsk branch of the Automatic Hardware Institute and in 1971 returned to ITC and headed the control system laboratory (later renamed in the laboratory of mathematical cybernetics) continuing the studies started at the discrete-programming laboratory. At that time the main scientific interests of V.S. Tanaev were focused on the development of

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the decompositional methods for solution of complex optimization problems. The results of these studies provided the foundation for his doctoral thesis "Parametric Decomposition of the Extremal Problems" which was successfully defended in 1977 at the Computer Center of the USSR Academy of Sciences. In 1980, V.S. Tanaev was entitled professor of mathematical cybernetics. During the 1970–1990s his students defended doctoral and candidate dissertations on the mathematical methods of optimization and the scheduling theory, and a scientific school was formed. In 1987 V.S. Tanaev was appointed the Acting Director, and in 1988, the Director of ITC (since 2002, the United Institute of Problems of Informatics, Belarussian National Academy of Sciences).

The organizational activity of V.S. Tanaev was multisided. He was the scientific advisor of the state programs of fundamental studies "Intellect" and "Infotekh," state scientific-and-technological program "Information Technologies," joint Belarus–Russia programs "Kosmos-BR" and "SKIF," Deputy Academic Secretary of the Department of Physics, Mathematics, and Informatics of the Belarussian NAS, Deputy Editor-in-Chief of the journal "Vestsi NAS Belarusi. Seryya Fizika-matematychnykh Navuk" (News of Belarus NAS. Series of Physical and Mathematical Sciences), Chairman of the Dissertation Council at the ITC of Belarus NAS, and member of the dissertation council at the Institute of Mathematics of Belarus NAS and Belarussian State University, President of the Belarus Society of Operations Research, Member of the Institution of Electrical Engineers. Development in the Belarus Republic of the supercomputer technologies and use of the information technologies in the space research is related with the name of Vyacheslav Sergeevich Tanaev. In 1994 he was elected the Corresponding Member, and in 2000, the Member of the Belarus NAS.

Academician of the Belarus NAS V.S. Tanaev made an substantial contribution to the solution of some problems of operations research. The results obtained were published in nine monographs [1–9], methodological manual [10], and 120 papers and reports. At the Belarus State University he read lectures on operations research, scheduling theory, and methods of optimization. He guided 18 candidates of science of which five later defended the doctoral dissertations. For his outstanding scientific achievement and fruitful pedagogical activity, V.S. Tanaev was awarded in 1995 the honorary title of the "Honored Scientist of the Belarus Republic." The State Prize of the Belarus Republic in the field of natural sciences was awarded in 1998 to V.S. Tanaev and a group of his followers, and in 2001 he was awarded the Prize of the Belarus National Academy of Sciences for the monograph "Teoriya raspisanii. Gruppovye tekhnologii" (Scheduling Theory. Group Technologies). He was awarded a Certificate of Honor of the Byelorussian Supreme Council (1981) and a medal of the International Academy of Informatization, Information Processes and Technologies (Moscow, 1999). The organization of "Space Veterans" (town of Korolev) in 2000 awarded V.S. Tanaev the honorary sign "For Mastering the Space" for his great contribution to the space research. The scientific works of V.S. Tanaev have influenced and are still influencing the development of the scheduling theory, operations research, mathematical programming, and other fields of the applied mathematics not only in Belarus, but also beyond its frontiers. We outline below the main scientific results of V.S. Tanaev.

2. SCHEDULING THEORY AND RELATED ISSUES

Since the early 1960s and to the late 1990s V.S. Tanaev actively worked in the field of the scheduling theory. Six of his nine monographs cover this topic. One of the first works of V.S. Tanaev was devoted to generalization of the well-known Bellman–Johnson problem of constructing the speed-optimal schedule of servicing n requests by two serial machines. Constructed was a function defined over the set of permutations, and an algorithm was developed to minimize it which has the same time complexity $O(n \log n)$ as the Johnson algorithm and allows one to solve both the Bellman–Johnson problem and its numerous generalizations.

The earlier works of V.S. Tanaev were concerned with the multi-stage servicing systems with the transfer operators moving the requests from one machine to another so as to provide the maximum system performance. The problem with a fixed number of the transfer operators and identical requests was reduced to seeking the optimum in the class of periodic schedules. A two-stage scheme of solution was suggested for the problem with unlimited number of operators and different requests. At the first stage, the problem is reduced to that of the traveling salesman with determination of the optimal permutations defining the schedule of request servicing by the machines. At the second stage, the minimal number of the transfer operators and their schedule are determined.

Since the early 1970s V.S. Tanaev paid much attention to the problems where the deadlines of request servicing are of great importance. Consideration was given to the problems of two types. In those of the first type, it is required to construct a schedule admissible in terms of the given deadlines. In the problems of the second type, a schedule minimizing the deadlines-dependent penalty function is sought. A method of branches and boundaries for minimization of the weighted number of delayed requests serviced by one machine and arriving simultaneously to the servicing system was proposed together with V.S. Gordon. For the case of nonsimultaneous arrival, considered were the properties of the optimal schedules, the polynomially solvable special cases of the problem were specified, and algorithms to solve them were proposed. For the one-stage system of identical machines, the necessary and sufficient conditions were established under which all requests can be serviced before expiration of the deadlines.

The class of problems where interrupts of the request servicing are admissible are of practical importance and are actively investigated until now. For the one-stage system of identical machines, V.S. Tanaev established sufficient conditions for existence of the optimal schedule without interrupts of the request servicing: the objective function must be nondecreasing and e-quasiconvex. For the problem of minimization of the maximal penalty at servicing by one machine of partially ordered requests arriving nonsimultaneously, polynomial algorithms were proposed under the assumption of the possibility of interrupts. If on the set of requests an arbitrary relation of preceding is defined on the set of requests, then the time complexity of the algorithm is equal to $O(n^2)$. For the tree relation, the complexity of the proposed algorithm is $O(n \log n)$.

Examination of the situations where a relation of preceding is defined on the set of requests occupies an important place in the works of V.S. Tanaev. One of the notable results (obtained together with Gordon) is as follows. For the problem of constructing a schedule minimizing the sum of exponential penalty functions, an algorithm of complexity $O(n \log n)$ was proposed, provided that the relation of preceding defined on the set of requests is representable as a set of input trees. At that, an auxiliary function playing the key part in the algorithm and called later the priority function was constructed for the aforementioned objective function.

Studies at the junction of various sections of discrete optimization are one of the distinctions of the scientific creativity of Vyacheslav Sergeevich. They are exemplified by the decomposition of finite sets into subsets and construction of the optimal permutations retaining the given grouping of the elements of a set.

In terms of the determinate servicing systems, the considered decomposition problems are treated as follows. A set of requests arrives in a fixed sequence to a system of m parallel machines that are ready for operation at different time instants. It is required to assign the requests to the machines so as to minimize their total cost of servicing. At that, the unit of time of operation of each machine has its own cost, and the cost of servicing each request is defined by the function depending on the instant of termination of its servicing. An algorithm of the time complexity $O(n^m)$ was proposed to solve this problem.

The need for search of the optimal permutation retaining the given grouping of elements arises, for example, at developing the structure of a servicing system modeling the shop of galvanic or chemical part processing. The technological sequences of the processing stages are the same for all parts. For technological reasons, the set of machines (bath with chemical solutions) corresponding to all m stages of processing is decomposed in several subsets. At that, one subset may include machines located at different parts of the technological sequence. Given are m linearly ordered positions where the machines may be situated. The machines of the first subset must occupy m_1 first positions, those of the second subset, the following m_2 positions, and so on, m_r being the cardinality of the rth subset. Needed is to determine the order of arrangement of the machines to positions for which the length of the request travel route is minimal. For this problem, an algorithm of the time complexity $O(m^2)$ was proposed in collaboration with A.K. Sannikova.

Some problems of discrete optimization, including those of the scheduling theory, may be formulated in terms of minimization of the symmetrical functions over an appropriate finite vector set. For the problems of this kind, V.S. Tanaev introduced the notion of the minorant vector set by which meant is the set having a single minimal vector in the sense of the relation $\leq$ at the coordinatewise comparison of vectors. The algorithm of minimization of the increasing symmetrical function on a minorant set was shown to minimize on this set any other nondecreasing symmetrical function. Whence it follows, in particular, that the polynomial solvability of the problem of minimization of an increasing symmetrical function on the given minorant vector set entails polynomial solvability of the problem of minimization of any nondecreasing symmetrical function on the same set. It was also established that the NP-hardness of the problem of minimization of some nondecreasing symmetrical function on the minorant set entails NP-hardness of the problem of minimization on this set of any increasing symmetrical function.

Since the mid-1960s V.S. Tanaev paid much attention to the mixed graphs which at that time were studied insufficiently. Later it turned out that namely the mixed graphs allowed one to construct adequate models of the multi-stage servicing systems and develop on this basis the "record" algorithms to construct the optimal schedules. The problem of constructing schedules permissible in terms of the relation of preceding defined on the operation set was reduced to that of constructing a set of circuit-free oriented graphs generated from the mixed graph by replacing its edges by arcs. At that, the arcs of the mixed define the constraints on the order of operations. The edge indicates to the impossibility of simultaneous execution of its incident operations (vertices). The schedule is defined by the circuit-free oriented graph obtained from the original mixed graph by replacing its edges by arcs. V.S. Tanaev established the upper analytical estimates of the cardinality of the set of the generated circuit-free oriented graphs for the complete mixed graph. Since in this case generated are the complete circuit-free graphs, the matter concerns listing of the permutations of the elements of partially ordered set. For an arbitrary mixed graph, algorithms were developed to calculate the lower estimate of the cardinality of the set of generated oriented graphs. An assertion was proved that for any two generated circuit-free oriented graphs G' and G'' it is possible to construct a sequence $G', G_1, \ldots, G_n, G''$ of the generated circuit-free oriented graphs differing from their neighbors in orientation of only one edge. This assertion enables one to construct such sequences of the oriented graphs if the initial and final oriented graphs of the sequence are given.

In 1976 V.S. Tanaev introduced the notion of coloring the vertices of the mixed graph by which is meant assignment of natural numbers (colors) to the mixed-graph vertices. The vertices connected by an edge must be different. The color number of the initial vertex of an arc should not exceed that of the terminal vertex of this arc. The necessary and sufficient conditions for existence of coloring of the mixed-graph vertices were established in collaboration with Yu.N. Sotskov.

The well-known Dilworth's problem is a special case of the problem of k-optimal decomposition of the partially ordered set into chains for $k = 1$. The Dilworth's theorem states that the minimal number of chains into which the partially ordered set can be decomposed is equal to the maximal number of pairwise incomparable elements of this set. In the problem of k-optimal decomposition, the function $f(k, C) = \min\{k, |C|\}$ is assigned to each chain C. A decomposition is called the k-optimal decomposition if for the given k the sum of values of the functions $f(k, C)$ with respect to all decomposition chains is minimal. V.S. Tanaev established the necessary and sufficient conditions for membership of an inclusion-maximal chain to the k-optimal decomposition.

3. PARAMETRIC DECOMPOSITION OF THE OPTIMIZATION PROBLEMS

The increased requirements on quality and speed of decision making in the socio-economic, industrial, and technical systems often constrain the possibility of direct use of the well-known "universal" methods of solution of complex optimization problems and give rise to the need for special methods allowing for the distinctions of the problems at hand. The decomposition-based approach proved to be promising for development of such methods. As early as in the 1970s, V.S. Tanaev took interest in the development of such approaches which represent a problem that is exceptionally important for practice and challenging for science.

During several years V.S. Tanaev developed in collaboration with G.M. Levin a general theory of parametric decomposition of the optimization problems. The theory relies on the concept of parametrization of the original problem which leads to its decomposition into a collection of hierarchically interrelated simpler subproblems. A general scheme of parametric decomposition, as well as the sufficient conditions for its application, were developed within the framework of this theory. The interrelations of the stationary domains and those of local minima of the objective functions of the resulting subproblems and the original problem **A** were considered under these conditions. Some causes were specified owing to which the stationary and local domains of the original problem **A** generate similar domains in the lower-level subproblem obtained as the result of decomposition. The stationary (local) domains in the coordinating upper-level subproblem **B** were classified in terms of their interrelations with similar domains of the original problem **A**. It was established that there are several types of such domains and only the domains of one type exert substantial influence on the complexity of solution of the subproblem **B**. It was proved that each stationary (local) domain of this type is generated in the subproblem **B** by the corresponding stationary (local) domain of the original problem **A**. Consequently, the number of local domains of this type in the subproblem **B** does not exceed the number of local domains in the original problem **A**.

The results obtained enabled purposeful development of efficient multi-level schemes of decomposition for solution of many classes of the optimization problems. Methods for solving some important applied problems of optimal decision making in the CAD systems of machine-building plants were developed on the basis of this theory at the ITC of Belarus NAS with participation of V.S. Tanaev.

The results obtained at this stage of studies were summarized in the monograph [7] that aroused considerable interest and in the reviews [11, 12]. With regard for importance of these studies, V.S. Tanaev was included in the Working Group on Decomposition and Coordination in Complex Systems established in 1988 at the Commission for using high-performance computers in scientific calculations of the USSR Academy of Sciences.

Wide use of the parametric decomposition at developing efficient methods of solution of the applied optimization problems of various classes brought to light advisability of using also some additional devices going beyond the formal scheme of parametric decomposition. This motivated the need to develop on the basis of earlier results the general theory of the so-called extended parametric decomposition using parametrization as a unique basis for complex application of the decomposition technique and immersion of the resulting subproblems into computationally simpler problems. This approach enabled one to combine the reasonable aspects of both techniques. This work was successfully carried out in the 1990s by V.S. Tanaev in collaboration with G.M. Levin and L.F. Verina [13]. The results obtained opened new opportunities in application of decomposition methods to solution of the complex optimization problems arising in the up-to-date decision-support systems in various knowledge domains.

4. OTHER DOMAINS OF THE APPLIED MATHEMATICS

Since the mid-1960s many countries began to develop actively the tabular languages as an efficient means for formal description of the decision making procedures in various situations. The applicability tables that were proposed in ITC AN BSSR proved to be a convenient tool for description of a wide range of CAD systems. V.S. Tanaev's attention was attracted to the problem of creating methods and means of automating design of efficient algorithms and programs realizing the process of decision making on the basis of such large-dimensionality tables. Together with M.P. Povarich, he considered in 1974–1975 the properties of the applicability tables and developed the methods to design the flowgraphs of the algorithms of decision making on the basis of such tables with minimization of the number of arcs in these graphs. Developed was a software package realizing these methods and enabling one to construct the flowgraphs, as well as to generate from them the texts of the decision-making programs written directly in the algorithmic languages such as ALGOL and FORTRAN. The results obtained were reflected in the monograph [9].

Much attention was paid by V.S. Tanaev to using applied mathematics and informatics in other knowledge domains. Substantial volume of research was carried out by him and his collaborators on the development of mathematical models and methods of CAD optimization. These developments were interrelated with the aforementioned works on the parametric decomposition of the optimization problems. Among the most interesting works in this field, special mentioning deserve works on optimization of the structure and parameters of the discrete technological processes of part processing on the multipositional equipment, including the works on optimization of the modes of multi-instrument processing on the multihead machines and automatic lines. It deserves noting that these works stimulated the studies of parametric decomposition of the optimization problems and at the same time relied on their results.

Practical application of the advances of the operations research to the development of the decision-support systems in particular knowledge domains implies availability of special software to solve a wide spectrum of "typical" optimization problems. Shortage of such means was felt strongly in the 1960–1980s. In these year, in his laboratory V.S. Tanaev initiated and headed development of such software complexes. In particular, he participated directly in the development of software packages for the optimization problems such as optimal ordering, convex programming with linear constraints, multi-step optimization of the recurrent monotone functions, polynomial programming, and minimization of the superposition of the recurrent monotone functionals on the paths of oriented graph. The works were carried out within the framework of the multifunction package of optimization programs of design, planning, and control created jointly by the experts of Belarus, Baltic, Ukraine, and Russia for the ES computer-based systems.

5. SYSTEMATIZATION OF THE RESULTS ON THE SCHEDULING THEORY AND DECOMPOSITION OF THE OPTIMIZATION PROBLEMS

Trying to visualize the creative activity of V.S. Tanaev, one cannot disregard his activity in systematization of the accumulated knowledge. This concerns first of all the scheduling theory. Vyacheslav Sergeevich devoted much time and efforts to writing monographs. His *Introduction to the Scheduling Theory* [1] (coauthored by V.V. Shkurba) published in 1975 by Nauka played an outstanding role in training the experts on the scheduling theory both in the USSR and some

Eastern-European countries. In 1984 and 1989 the same publishing house published the monographs [2] (coauthored by Gordon and Ya.M. Shafransky) and [3] (coauthored by Sotskov and V.A. Strusevich) describing actually all most important results concerning the deterministic servicing systems that were known by that time. The monographs [2, 3] summarized in a way the thirty-years of the history of the scheduling theory. They are complemented by the paper [14] which classified and reviewed the approximate methods to solve the problems of optimal scheduling. In 1994 Kluwer Academic Publishers translated into English the reworked and complemented versions of these books [4, 5], thus making accessible to the Easters readership the results that previously existed only in Russian. The last (1998) in this series of monographs [6] (coauthored by M.Ya. Kovalyov and Shafransky) was devoted to a comparatively new direction in the scheduling theory, the group technologies.

V.S. Tanaev published two monographs [7] (coauthored by G.M. Levin) and [8] in Nauka i Tekhnika (Minsk) on decomposition of the optimization problems. The first one (1978) presents a sufficiently full exposition of the results of V.S. Tanaev and his colleagues on the parametric decomposition of the optimization problems and some generalizations of this method. The decompositional methods of solution of some applied problems that were developed on the basis of parametric decomposition were discussed in detail. The second monograph (1987) was devoted to describing the main approaches to constructing the decomposition procedures for solution of the large-dimensionality mathematical programming problems. V.S. Tanaev maintained permanent contacts with Belarus State University, the greatest higher education institution, read there courses of lectures, and organized at ITC of AN BSSR a branch of the university chair. The special course on the decomposition methods to solve the optimization problems that he read at the University in the early 1980s underlies the monograph [8]. In 1994 the "Universitetskoe" publishers (Minsk) published a textbook [10] (coauthored by Sotskov and Strusevich) on scheduling. Being involved in teaching, Vyacheslav Sergeevich was well aware of the need for knowledge popularization. In 1988 the "Znanie" publishers (Moscow) published his booklet "Scheduling Theory" [15]. V.S. Tanaev's desired to unify the terminology and notation, generalize and present various results from the same methodological standpoint is the distinctive feature of the aforementioned monographs. We conclude by noting that the scientific results obtained by V.S. Tanaev exerted and continue to exert great influence on the development of some lines of scientific research. His school of the scheduling theory and parametric decomposition is recognized by the scientific society. Several V.S. Tanaev's monographs became the reference books of an entire generation of experts. Beginning from 2003, an International Conference "Tanaev Readings" contemporized to the V.S. Tanaev's birthday is held biannually (in 2009 it was moved to 2010 because of the jubilee). Its aim is to promote the fundamental and applied studies in the field of the scheduling theory, decomposition of complicated problems, optimization, planning, and production control. The list of conference topics is continuously extending and includes now other promising lines of the information technologies.

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