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Vyacheslav Tanaev: contributions to scheduling and related areas

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Abstract The paper discusses several areas of research conducted by Vyacheslav Tanaev (1940–2002), mainly on scheduling. His contribution to parametric decomposition of optimization problems is also addressed. For each area we focus on the most important results obtained by V.S. Tanaev and trace how his research has been further advanced.

Keywords Scheduling · Sequencing · Permutation · Priority-generating function · Symmetric function · Mixed graph · Parametric decomposition

1 Introduction

Vyacheslav S. Tanaev was born on March 28, 1940, in Akulovo, Tver region, Russian Federation. He obtained his high education from the Crimea Pedagogical Institute in 1962, and received his Candidate of Sciences (PhD Equivalent) degree for his work on scheduling from the Institute of Mathematics of the National Academy of Sciences of Belarus (NASB) in 1965. The degree of Doctor of Sciences (Habilitated Doctor) was awarded to V.S. Tanaev after a successful defense of the thesis on parametric decomposition of optimization problems at the Computer Center of the Academy of Sciences of the USSR, Moscow, in 1977. In 1963 Vyacheslav Tanaev started to work at the NASB, sequentially taking positions of a PhD student, a researcher and a head of a laboratory. In 1987 he became the director of the Institute of Engineering Cybernetics (United Institute of Informatics Problems of NASB since 2002), and in 2000 was elected a full member of NASB, which is the highest scientific rank in the states of the former Soviet Union. The scientific heritage of Vyacheslav Tanaev includes more than 130 research publications among which there are ten monographs. His scientific interests included scheduling theory, discrete and continuous optimization, computer aided design; he coordinated research in geo-information systems, development of super-computers, application of informatics to medicine. He supervised 18 Candidates of Sciences among which 6 became Doctors of Sciences. The authors of this review are former students and colleagues of Vyacheslav Tanaev, and are greatly indebted to him for showing the right way of their scientific career.

It is well-known that Scheduling Theory as a separate branch of Operational Research started in the middle of 1950s. During its first decade, Scheduling Theory was mainly developed in the USA, see Potts and Strusevich (2009) for a review. A time lag of almost ten years had elapsed before the first papers on scheduling appeared in other countries. Presumably, Vyacheslav Tanaev is the author of the first papers in Russian with "scheduling" in the title (Tanaev 1964a, 1964b, 1964c), and his early research

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stimulated the study in this area in the Soviet Union and the countries of Eastern Europe. Later, when strong scheduling groups appeared in all major scientific centers of the USSR (Minsk, Moscow, Kiev, Novosibirsk, etc.), till his last days he remained the main authority in the area. Several generations of Russian-speaking researches benefited from getting familiar with major results on scheduling by studying the monographs (Tanaev and Shkurba 1975; Tanaev et al. 1984a, 1989a, 1989b); the two latter books appeared in English in 1994.

In this paper, we have selected several topics of research initiated by V.S. Tanaev and traced how his results have been further extended and developed.

It is assumed that the reader is familiar with the scheduling terminology and the three-field classification system for scheduling problems, as introduced in Graham et al. (1979).

2 From simple priorities to scheduling with precedence constraints

In the early days of scheduling research, most of the studies focused on combinatorial analysis of the relevant models. It was found that quite often a mathematical model of a scheduling situation could be formulated in terms of optimizing a function over a set of permutations of jobs. The first scheduling results proved in the middle of 1950s for the two-machine flow shop problem to minimize the makespan (Johnson 1954), for the single machine problem to minimize the maximum lateness (Jackson 1955), for the single machine problem to minimize the sum of weighted completion time (Smith 1956) were obtained by the so-called pairwise interchange argument that could be traced back to the famous book (Hardy et al. 1934). As a result, the mentioned problems admit a solution by a so-called priority rule: each job is assigned a value (that depends only on the job's parameters), called a priority, and an optimal sequence is obtained by sorting the jobs according to these priorities. As a proof technique, the pairwise interchange argument uses the following reasoning. Suppose that there exists an optimal sequence in which some two adjacent jobs do not follow the rule under consideration. Then a proof is provided that if the order of these jobs is reversed, the objective function value does not get worse, thus ordering the jobs according to the rule is a sufficient condition for optimality of the obtained sequence.

In this section, we discuss how the technique of minimizing functions over permutations of independent elements (jobs) using simple job priorities has been advanced to minimization of functions of a certain type over partially ordered sets using extended priorities assigned to sequences of jobs. As an illustration, throughout this section we use the function similar to that studied in Smith (1956) and its generalizations.

Given a set $N=\{1,2,\dots,n\}$ of jobs, let $\pi_r=(j_1,j_2,\dots,j_r)$ denote a permutation of r jobs selected from N, where $0\le r\le n$. Here r is the length of permutation π_r; the case of $r=0$ corresponds to a dummy permutation. Denote by $\hat{\mathcal{P}}_n$ the set of all full permutations, i.e., permutations of length n. The early scheduling results mentioned in the opening paragraph of this section can be seen as related to minimizing a function $F(\pi)$ over the set $\hat{\mathcal{P}}_n$. The pairwise interchange argument is applicable if a job $j\in N$ can be associated with a value $\omega(j)$ such that for any two jobs j and k the inequality $\omega(j)\ge\omega(k)$ implies that there exists an optimal permutation in which job j is scheduled before job k. An alternative interpretation: if $\pi=(j_1,j_2,\dots,j_k,j_{k+1},\dots,j_n)$ and $\pi'=(j_1,j_2,\dots,j_{k+1},j_k,\dots,j_n)$ are two full permutations that differ only by the transposition of the jobs sequenced in the kth and $(k+1)$th positions and $\omega(j_k)\ge\omega(j_{k+1})$ then $F(\pi)\le F(\pi')$. In what follows, we call the function $\omega(j)$ that satisfies the outlined property the *1-priority function*.

For example, for the problem studied in Smith (1956), the objective function can be written as

$$F(\pi)=\sum_{k=1}^{n} w_{j_k}C_{j_k}=\sum_{k=1}^{n} w_{j_k}\sum_{i=1}^{k} p_{j_i}, \qquad (1)$$

where p_{j_k}, C_{j_k} and w_{j_k} denote the processing time, the completion time and the weight of job j_k sequenced in position k, for $1\le k\le n$. It is shown in Smith (1956) that we can select the 1-priority $\omega(j)=w_j/p_j$ and sort the jobs in non-increasing order of these values.

In one of his early papers Tanaev introduces a generalization of the function of the form (1). He defines

$$F(\pi)=\sum_{k=1}^{n}\varphi_{j_k}\left(\sum_{i=1}^{k} p_{j_i}\right), \qquad (2)$$

where $\varphi_j(t)$ is a function of job completion times (Tanaev 1965) and shows that for $\varphi_j(t)=\varphi(t)$, where $\varphi(t)$ is an arbitrary non-decreasing function, and for $\varphi_j(t)=\alpha_j\exp(\gamma t)$, where $\gamma\ne 0$, the function of the form (2) admits 1-priorities $\omega(j)=-p_j$ and $\omega(j)=\alpha_j\exp(\gamma p_j)/(\exp(\gamma p_j)-1)$, respectively. A similar result was independently derived by Rothkopf (1966). Several years later, Kladov and Livshitz (1968) obtained the result that can be interpreted as follows. Function (2) defined in terms of non-decreasing and sufficiently smooth functions φ_j admits a 1-priority if and only if (i) $\varphi_j(t)=\alpha_j t+\beta_j$ or (ii) $\varphi_j(t)=\alpha_j\exp(\gamma t)+\beta_j$ or (iii) $\varphi_j(t)=\varphi(t)+\beta_j$, where $\varphi(t)$ is an arbitrary non-decreasing function.

All results mentioned above are related to minimizing certain objective functions over the set $\hat{\mathcal{P}}_n$ of all full permutations, i.e., the jobs are *independent* and any job may take any position in a permutation that defines a schedule. However, it is often found in practice that not all permutations of

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jobs are permitted due to various technological, marketing or assembly requirements. This can be modeled by imposing *precedence constraints* on set N to describe allowable sequences of jobs, according to which only permutations of a certain set $\mathcal{P} \subset \hat{\mathcal{P}}_n$ are feasible.

Formally, the precedence constraints are defined by a binary relation $\rightarrow$. We write $i \rightarrow j$ and say that job i *precedes* job j if in any feasible schedule job i must be completed before job j starts its processing. In the case of multi-stage systems, such as the flow shop, it is required that job i must be completed on any machine before job j starts on that machine. The set of constraints is usually given by an acyclic directed graph G, in which the set of vertices corresponds to the set of jobs N and there is an arc from vertex i to vertex j if and only if $i \rightarrow j$. It is convenient to represent the constraints in the form of a *reduction graph*, obtained from G by removing all transitive arcs. A permutation of jobs is *feasible* if no pair of jobs violates the precedence constraints. Thus, set N together with the defined precedence relation $\rightarrow$ should be seen as a *partially ordered set*, or a *poset*. Let $\mathcal{P}_n(G)$ denote the set of all full feasible permutations of a partially ordered set of n elements.

V.S. Tanaev was among the first who understood the importance of scheduling problems under precedence constraints. In 1967 he wrote an elegant one page long paper (Tanaev 1967) on enumeration of feasible permutations of a poset.

In the monograph (Conway et al. 1967) the single machine problem to minimize the sum of the job completion times under chain-like precedence constraints was considered and the authors came up with an idea of considering not individual jobs, but sequences of jobs called composite jobs later. This idea has been independently extended to minimizing a function of the form (2) under tree-like precedence constraints in Horn (1972) for $\varphi_j(t) = \alpha_j t + \beta_j$ and in Gordon and Tanaev (1973a) for $\varphi_j(t) = \alpha_j \exp(\gamma t) + \beta_j$.

It is beyond the scope of this paper to give a detailed historical account of scheduling under precedence constraints. Among those who made essential contributions by generalizing simple job priorities in the 1970s are D.L. Adolphson, V.S. Gordon, E.L. Lawler, T. Kurisu, C.L. Monma, Y.M. Shafransky, J.B. Sidney and many others. Briefly, the main results obtained in this area can be summarized as follows: For certain objective functions an optimal permutation in set $\mathcal{P}_n(G)$ can be found in polynomial time if the reduction graph G is series-parallel. The objective functions that allow this are now known as *priority-generating*. This concept was first introduced in Shafransky (1978a) and provisionally reported in Gordon and Shafransky (1977); the theory of minimization of such functions was further developed in Y.M. Shafransky's PhD thesis and in Gordon and Shafransky (1978a, 1978b, 1978c). A good review of the related issues is given in Tanaev et al. (1989a); see also Monma and Sidney (1979) who independently used similar concepts, e.g., the *adjacent sequence interchange property*. Normally, the running time of the resulting algorithms of minimizing a priority-generating function over set $\mathcal{P}_n(G)$ is $O(n \log n)$, provided that a series-parallel graph G is given by its decomposition tree. A systematic exposition of the theory of minimization of priority-generating functions over series-parallel and more general precedence constraints is included as a chapter into the monograph (Tanaev et al. 1984a).

The formal definition of a priority-generating function and the priority function is given below. Let us be given a set $\mathcal{P} \subset \hat{\mathcal{P}}_n$ of feasible permutations (in the case of a poset, e.g., $\mathcal{P} = \mathcal{P}_n(G)$). Denote by $Q[\mathcal{P}]$ the set of all substrings of permutations from $\mathcal{P}$, i.e., $\pi^0 \in Q[\mathcal{P}]$ if there exist partial permutations π_1 and π_2 such that $(\pi_1, \pi^0, \pi_2) \in \mathcal{P}$. Let $\pi^{\alpha\beta} = (\pi', \alpha, \beta, \pi'')$ and $\pi^{\beta\alpha} = (\pi', \beta, \alpha, \pi'')$ be two feasible permutations that differ only in the order of the substrings α and β. For a function $F(\pi)$, suppose that there exists a function $\omega(\pi)$ defined over the set $Q[\mathcal{P}]$ such that for any two feasible permutations $\pi^{\alpha\beta}$ and $\pi^{\beta\alpha}$ the inequality $\omega(\alpha) \geq \omega(\beta)$ implies that $F(\pi^{\alpha\beta}) \leq F(\pi^{\beta\alpha})$. In this case, function F is called a *priority-generating function* over set $\mathcal{P}$, while function ω is called its *priority function*. For a (partial) permutation π, the value of $\omega(\pi)$ is called the *priority* of π.

A result presented in Zinder (1976) can be interpreted as follows: function (2) is priority-generating over $\hat{\mathcal{P}}_n$ if and only if (i) $\varphi_j(t) = \alpha_j t + \beta_j$ or (ii) $\varphi_j(t) = \alpha_j \exp(\gamma t) + \beta_j$. In the case (i) the priority function is

$$\omega(\pi) = \sum \alpha_j \Big/ \sum p_j,$$

while in the case (ii) it is

$$\omega(\pi) = \Big(F(\pi) - \sum \beta_j\Big) \Big/ \Big(\exp\Big(\gamma \sum p_j\Big) - 1\Big).$$

Here and below we assume that all summations are taken with respect to the jobs included into a partial permutation π. These and many other priority-generating functions and their priority functions can be found in Tanaev et al. (1984a).

Apart from the function (2), another important function in scheduling is

$$F(\pi) = \max_{1 \leq u \leq n} \left\{ \sum_{k=1}^{u} \alpha_{j_k} + \beta_{j_u} \right\}, \tag{3}$$

closely related to the minimization of makespan in the two-machine flow shop (Johnson 1954). This function in the form (3) is introduced in Tanaev (1964c) and can serve as a unified model for most versions of the two-machine flow shop problem that involve various additional time lags (setup times, transportation delays, etc.). When minimized

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over set $\hat{\mathcal{P}}_n$ of all full permutations, this function admits a 1-priority $\omega(j) = \operatorname{sgn}(-\alpha_j)(W - \min\{\alpha_j + \beta_j, \beta_j\})$, where $\operatorname{sgn}(x) = 1, 0, -1$ for $x > 0$, $x = 0$ and $x < 0$, respectively, and W is a sufficiently large number, greater than $\max_{j\in N} \min\{\alpha_j + \beta_j, \beta_j\}$; see Tanaev (1964c), Tanaev and Shkurba (1975). This function is also priority-generating, and its priority for a feasible (partial) sequence π is given by

$$\omega(\pi) = \operatorname{sgn}\Big(-\sum \alpha_j\Big)\Big(W - F(\pi) + \max\Big\{\sum \alpha_j, 0\Big\}\Big).$$

In this form this priority function is derived in Shafransky (1978b), see also Tanaev et al. (1984a).

Applications of the theory of minimization of priority-generating functions to multicriteria sequencing and scheduling are presented in Tuzikov and Shafransky (1983), Kovalyov and Tuzikov (1994), Janiak et al. (2001), to group technology scheduling are detailed in the monograph (Tanaev et al. 1998); to scheduling problems with due date assignments are considered in the papers of Gordon and Strusevich (1999) and Gordon et al. (2005); to scheduling problems with the start-time dependent and position dependent processing times are studied in Gordon et al. (2008).

We conclude this subsection by mentioning several results on scheduling problems with precedence constraints that are not necessarily solvable by priority-generation. Problems of this type have also been in the focus of attention of V.S. Tanaev and his team.

An $O(n^2)$-time algorithm is presented in Gordon and Tanaev (1983) for problem $1|r_j, prec, pmtn| \max\{f_j(C_j)\}$, subject to the condition that the value of each function $f_j(C_j)$ can be calculated in a constant time. The number of preemptions in an optimal schedule obtained by the algorithm is at most $n-1$. Some special cases are presented where the algorithm gives an optimal non-preemptive schedule (for example, when $p_j = d$, $j = 1, \ldots, n$, where d is the greatest common devisor of $r_1, \ldots, r_n$). Notice that a similar approach to solving the problem is independently proposed in Baker et al. (1983), and a special case of the maximum lateness minimization is similarly handled in Blazewicz (1976), see also Blazewicz et al. (2007).

A quite general model for scheduling on m unrelated parallel machines with a linearly ordered set of jobs is studied in Tanaev (1979a). Here the machines may have the ready times before which they are not available, and the sequence of jobs assigned to a machine must respect the linear order. Each job $j \in N$ is associated with a penalty $f_j(C_j)$, where f_j is a non-decreasing function of the completion time C_j. There is another cost component $F(V_1, \ldots, V_m)$, where F is a non-decreasing function of machine completion times $V_1, \ldots, V_m$. Additionally, each machine M_i has a cost function of its usage, i.e., to process a job j on machine M_i costs c_{ij}. The objective is to minimize the total cost as the sum of three components: $\sum_{j\in N} f_j(C_j)$, $F(V_1, \ldots, V_m)$ and the total cost of using the machines. An $O(n^m)$-time dynamic programming algorithm is suggested based on partition of the (ordered) set of jobs into subsequences.

A fairly complete complexity classification of shop scheduling problems under precedence constraints is given in Strusevich (1997a, 1997b), while scheduling problems with machine-dependent precedence constraints are studied in Shafransky and Strusevich (1998), Gladky et al. (2004).

3 Mixed graphs and multigraphs in scheduling theory

One of the most general and most difficult to handle scheduling models is the job shop, traditionally denoted by $J\|F$, where F is a (regular) objective function to be minimized. Here the jobs have to be processed sequentially on a number of machines, each job has its individual processing route, in which some of the machines can be missing, some can be repeated several times (revisited). Even in the case of three machines and three jobs problem $J3|n=3|F$ is binary NP-hard for all traditional objective functions F, as shown in Sotskov and Shakhlevich (1995).

In the middle of the 1960s, several researchers came to an idea of modeling the job shop problem in terms of either obtaining a circuit-free digraph from a so-called disjunctive graph (Roy and Sussmann 1964) or, equivalently, finding a circuit-free orientation of the edges of a weighted mixed graph. The latter model was introduced by V.S. Tanaev and has remained more typical in the East European literature.

V.S. Tanaev initiated study of an extremal problem on mixed graphs as a model of a general shop, i.e., a multi-stage scheduling system that generalizes the traditional job shop, open shop and mixed shop. A general shop problem, which we here denote by $G\|F$, can be represented by means of a weighted mixed graph $G = (Q, C, D)$. Here Q is a set of vertices (operations), a non-negative weight (the sum of operation durations, setup time and transfer time, if any) being assigned to each vertex $i \in Q$. The set C of arcs represents the given precedence constraints. The set D of edges represents the competition between operations, which either have to be processed on the same machine or belong to the same job processed without a fixed machine route like in an open shop. A pair of non-negative weights is assigned to each edge. Using the mixed graph approach, one can consider problem $G\|F$ as an extremal problem on a weighted mixed graph (Matyushkov and Tanaev 1967, 1968; Tanaev 1975, 1988): To find the orientation of each edge of set D such that the obtained digraph $G' = (Q, C \cup D', \emptyset)$ has no circuits and the objective function F achieves its minimum value for the semi-active schedule defined by digraph G'. Let $\Pi(G)$ denote the set of all circuit-free digraphs $G' = (Q, C \cup D', \emptyset)$ generated by a mixed graph G. The modelling of problem

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$G\|F$ in terms of mixed graphs is based on the one-to-one correspondence between set $\Pi(G)$ and the set of all semi-active schedules feasible for problem $G\|F$.

It is shown in Lambin and Tanaev (1970) that for a given mixed graph G and two digraphs $G_1 \in \Pi(G)$ and $G_k \in \Pi(G)$ one can produce a sequence of digraphs $G_1, G_2, \ldots, G_k$, in which one circuit-free digraph is translated into another, so that each next digraph of this sequence is obtained from the previous one by changing the orientation of exactly one arc. Algorithms for generating set $\Pi(G)$, for computing the exact value of $|\Pi(G)|$, and for finding lower bounds and upper bounds on $|\Pi(G)|$ are presented in Sotskov and Tanaev (1974, 1976a). The above results are included in the monograph (Tanaev et al. 1989b) and the textbook (Sotskov et al. 1994).

For the problem of minimizing the makespan, i.e., for $F = C_{\max}$, an optimal digraph G' that defines an optimal schedule has the minimum critical path. The algorithms and software for generating set $\Pi(G)$ were developed in Matyushkov and Tanaev (1967, 1968), with an emphasize on priority rules for selecting heuristic solutions of problem $G\|C_{\max}$. The information about the successful decisions, which led to good solutions, was accumulated and used for constructing more complex, adaptive priority rules. Such an adaptive approach was further developed in Shakhlevich et al. (1996), Kruger et al. (1998) in order to produce, for a class of similar problems $G\|C_{\max}$, a class-specific heuristic rule which would be successful for solving problems in this class.

Based on the concept of a stability radius introduced in Leont'ev (1975) for the traveling salesman problem, V.S. Tanaev has encouraged research on stability analysis of the optimal digraph $G' = (Q, C \cup D', \emptyset)$. Here the main question is to find the range of changes in operation durations that leave digraph G' optimal. A closed ball in the space of operation durations (with respect to the Chebyshev metric) is called a *stability ball* of G' if digraph G' remains optimal for any choice of operation durations from this ball. The maximum value of the radius of a stability ball is called the *stability radius* of G'. In Sotskov and Alyushkevich (1988), Alyushkevich and Sotskov (1989), Sotskov (1991), Sotskov et al. (1997, 1998b) the formulas for calculating the stability radius of the optimal digraph that defines an optimal semi-active schedule for problems $G\|C_{\max}$ and $G\|\sum C_i$ are obtained; the necessary and sufficient conditions for the stability radius to be equal to zero are given, and the class of optimal schedules with an infinitely large stability radius is discovered. The surveys of the results on the stability radius are given in Sotskov et al. (1995, 1998a), Emelichev et al. (2002); the relevant material is included into the monograph (Tanaev et al. 1989b) and the textbook (Sotskov et al. 1994).

In the monograph (Tanaev et al. 1989b), a resource-constrained project scheduling problem (RCPSP) is presented as an extremal problem on a weighted mixed multigraph $\overline{G} = (Q, C, \overline{D})$ without a restriction that the weights of arcs and edges must be non-negative. The schedule for RCPSP is defined by a multigraph $\overline{G}' = (Q, C \cup \overline{D}', \emptyset)$ with no circuit of a positive weight obtained from $\overline{G}$ due to orientation of each edge. In Sotskov and Tanaev (1989), it is proved that testing the existence of digraph $\overline{G}'$ is a strongly NP-hard problem even if there exists only one negative weight, and polynomially solvable cases of the latter problem are classified.

In the monograph (Tanaev et al. 1989b), optimization of a processing system is presented using a weighted mixed multigraph $\overline{G} = (Q, C, \overline{D})$. The optimization problem P includes (a) choosing machines from given sets of machines of different types, (b) assigning the given set of operations to the chosen machines, and (c) sequencing the operations in accordance with the assignment. In Sotskov (1997), Sotskov et al. (2002), it is shown that steps (a), (b) and (c) may be carried out simultaneously due to special transformations of the edges of set $\overline{D}$ in the mixed multigraph $\overline{G}$. The necessary and sufficient conditions for a digraph generated by a mixed multigraph $\overline{G}$ to define a feasible solution for problem P are proved.

It should be noted that most of the papers written and co-authored by V.S. Tanaev before 1990 were published in Russian and therefore were almost inaccessible to scientists in the West. For example, V.S. Tanaev initiated the study of mixed graph coloring, i.e., assignment of positive integers (colors) to vertices of a mixed graph so that, if two vertices are linked by an edge then their colors have to be different, and if two vertices are linked by an arc, then the color of the start-vertex has to be no greater than the color of the end-vertex. There is a more than 20 years long gap between the first paper on mixed graph coloring published in Russian (Sotskov and Tanaev 1976b) and the first paper on this topic that appeared in English (Hansen et al. 1997). For a mixed graph, bounds on the chromatic number, i.e., on the smallest integer k for which a mixed graph admits a coloring in k colors) are presented in Ries and de Werra (2008). The complexity status of finding the chromatic number for a mixed graph is studied in Ries (2007). In particular, it is proved that the problem is NP-hard for planar bipartite mixed graphs and for bipartite mixed graphs with a degree at most 3. Mixed graph coloring can be interpreted as a scheduling problem $G|p_{ij} = 1|C_{\max}$ with unit processing times (Sotskov et al. 2001, 2002). In Sotskov and Tanaev (1976b), Sotskov et al. (2002) the chromatic polynomial of a mixed graph is studied. Such a polynomial may be used for calculating the number of feasible schedules. In Sotskov et al. (2001) and Al-Anzi et al. (2006), the problems $J|p_{ij} = 1|C_{\max}$ and $J|p_{ij} = 1|\sum C_i$ are considered in terms of mixed graph coloring; the complexity results are proved for special cases and branch-and-bound algorithms for mixed graph coloring are developed and tested.

4 Functions in scheduling

V.S. Tanaev was deeply interested in the mathematical background of scheduling results, in particular in determining classes of objective functions that will guarantee certain properties of an optimal schedule and/or of a solution algorithm. We have already discussed how Tanaev's results initiated the developments of the theory of minimization of priority-generating functions; see Sect. 2. In this section, we discuss further examples of classes of functions studied or introduced by Tanaev.

4.1 Non-preemptive schedules and *e*-quasi-concave functions

One of the early results of scheduling theory obtained in McNaughton (1959) establishes that for the single machine problem $1|pmtn|\sum f_j(C_j)$ to minimize the sum of non-decreasing penalty functions f_j, there exists an optimal non-preemptive schedule. According to the study conducted by Gordon and Tanaev in the early 1970s, this is in fact valid for any non-decreasing objective function of the completion times of the jobs, i.e., for an arbitrary function $F(\overline{x}) = F(x_1, \ldots, x_n)$, where $\overline{x}$ is an n-vector with $x_j = C_j$ that does not decrease in each of its arguments. Moreover, if the jobs have different release dates r_j, then for the resulting problem $1|r_j, pmtn|F$ there exists an optimal schedule in which preemptions can occur only at the release dates. See Sect. 1 of Chap. 4 of Tanaev and Shkurba (1975) and Sect. 1 of Chap. 2 of Tanaev et al. (1984a) for details, proofs and a historical account.

To describe the class of problems on parallel identical machines for which there exists an optimal non-preemptive schedule, Tanaev introduces the notion of an e-quasi-concave objective function. Recall that a function $F(\overline{x})$ is *quasi-concave* if for any n-vectors $\overline{x}^{(1)}$ and $\overline{x}^{(2)}$ and an arbitrary λ, $0 \le \lambda \le 1$, the inequality

$$F\big(\lambda\overline{x}^{(1)} + (1-\lambda)\overline{x}^{(2)}\big) \ \ge\ \min\big\{F\big(\overline{x}^{(1)}\big), F\big(\overline{x}^{(2)}\big)\big\} \qquad (4)$$

holds. Let E be the set of all n-vectors $\overline{e}$ with the components $0, 1$ and -1. A function $F(\overline{x})$ is called *e-quasi-concave*, if for any n-vectors $\overline{x}^{(1)}$ and $\overline{e} \in E$ and any numbers α and λ, $\alpha > 0$, $0 \le \lambda \le 1$, inequality (4) holds for $\overline{x}^{(2)} = \overline{x}^{(1)} + \alpha\overline{e}$; see Sect. 1 of Chap. 2 of the monograph (Tanaev et al. 1984a). Notice that another equivalent definition of an e-quasi-concave function is given in Tanaev (1973). By definition, a concave function is quasi-concave, and a quasi-concave function is e-quasi-concave, but not vice versa. It is proved in Tanaev (1973) (see also Sect. 1 of Chap. 4 of Tanaev and Shkurba 1975 and Sect. 1 of Chap. 2 of Tanaev et al. 1984a) that for problem $P|pmtn|F$ with a non-decreasing and e-quasi-concave objective function there exists an optimal non-preemptive schedule. Moreover, instances of problem $P|pmtn|F$ are presented for which a non-preemptive optimal schedule does not exist if either the objective function is not e-quasi-concave or the jobs are partially ordered.

In Gordon and Tanaev (1973b), conditions on the objective function (more general than the property of being e-quasi-concave) are established such that for problem $P|r_j, pmtn|F$ there exists an optimal schedule in which preemptions occur only at the release dates. Besides, problem $P|r_j, pmtn, C_j \le d_j|\cdot$ of finding a deadline-feasible preemptive schedule on parallel identical machines is considered in Gordon and Tanaev (1973c); see also Chap. 2 in Tanaev et al. (1984a). The necessary and sufficient conditions of the existence of such a schedule are formulated by reducing this scheduling problem to a maximum flow problem. Notice that a similar approach has been independently developed in Horn (1974).

For the single machine scheduling problem of minimizing the weighted number of late jobs, it is shown in Tanaev and Gordon (1983) that there exists a non-preemptive optimal schedule, provided that the release dates and the due dates are similarly ordered, i.e., $r_i < r_j \Rightarrow d_i \le d_j$ for all $i, j \in N$. The conditions under which an optimal sequence of jobs can be found in $O(n^2)$ time are given. These conditions are valid, in particular, for the unweighted case. If the release dates, the due dates, the weights and the processing times are agreeable, i.e., the jobs can be numbered as either (i) $r_j \le r_{j+1}, d_j \le d_{j+1}, p_j \le p_{j+1}, w_j \ge w_{j+1}$ or (ii) $r_j \le r_{j+1}, d_j \le d_{j+1}, p_j \le r_{j+1} - r_j, w_j \ge w_{j+1}$, then an optimal non-preemptive schedule exists and can be found in $O(n \log n)$ time.

4.2 Optimization of recursive functions over a set of permutations

Many applied problems, including those of scheduling theory, can be formulated in terms of optimization of certain functions over subsets of permutations of a finite set of elements (jobs). In the 1960s and 1970s V.S. Tanaev together with G.M. Levin identified and studied one of fairly general classes of such problems, namely, problems of optimization of monotone-recursive functions over normalized sets of permutations.

This direction of research started with considering a single machine scheduling problem to minimize the makespan under arbitrary precedence constraints, provided that the completion time of a job sequenced in the kth position depends on the completion time of the job in the previous position $k-1$ and additionally on the jobs (not necessarily on their completion times only) sequenced in several preceding positions. The processing time of a job depends both on this job and its position in the sequence. In Tanaev and Levin (1967) a solution approach based on dynamic programming

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(DP) and branch-and-bound (BandB) is outlined. A similar model is considered in Levin and Tanaev (1968), however here the completion time of a job in the kth position additionally depends on the jobs that are sequenced after that position.

A more general situation is considered in Tanaev (1977a). The problem is to minimize a function $g(\pi) = F(\pi, n)$ for a function F that is recursively defined over a set of permutations of a partially ordered set of n elements by the formula $F(\pi, k) = \Phi(F(\pi, k-1), \{\sigma\}, j_k)$, where $\pi = (j_1, \ldots, j_n)$ is a full permutation, σ is a partial permutation $\sigma = (j_1, \ldots, j_{k-1})$ and $\{\sigma\}$ represents the set of the elements in σ. The conditions are established under which the problem can be solved by an efficient algorithm that combines the DP and B&B ideas.

The next level of abstraction and generalization has led to optimization of monotone-recursive functions over normalized sets of permutations; see Levin and Tanaev (1970, 1978). The relevant concepts are presented and discussed below.

Let $\mathcal{P}$ be a set of (partial) permutations of the form $\pi_i = (j_1, j_2, \ldots, j_{\ell_i})$ of the elements of a finite set $N = \{1, 2, \ldots, n\}$. Define the set of ordered pairs $R = \{\langle\pi_i, k\rangle \mid \pi_i \in \mathcal{P}, 1 \le k \le \ell_i\}$. The main problem under consideration is to minimize the function $g(\pi_i) = f(\langle\pi_i, \ell_i\rangle)$ which is recursively defined over set $\mathcal{P}$ as

$$f\big(\langle\pi_i, k\rangle\big) = \Phi\big(f\big(\langle\pi_i, k-1\rangle\big), r\big(\langle\pi_i, k\rangle\big)\big),$$
$$k = 1, \ldots, \ell_i, \qquad (5)$$

where $f(\langle\pi_i, 0\rangle) = const$, and $r(\langle\pi_i, k\rangle)$ is a set defined below.

Tanaev and Levin study the case that set R is partitioned into non-empty mutually disjoint sets R_p, i.e., $R = \cup R_p$; and in turn, each of these sets R_p is also partitioned into non-empty mutually disjoint sets r_{pq}, i.e., $R_p = \cup r_{pq}$. Let $R(\langle\pi_i, k\rangle)$ (respectively, $r(\langle\pi_i, k\rangle)$) denote the set from the partition $\{R_p\} = \{R_p | p = 1, \ldots\}$ (respectively, from the partition $\{r_{pq}\} = \{r_{pq} | p = 1, \ldots; q = 1, \ldots\}$) that contains the element $\langle\pi_i, k\rangle$. Non-strict order relations $\Rightarrow$ and $\rightarrow$ can be defined over the set $\{R_p\}$ and over the set $\{r_{pq}\}$, respectively, which are coordinated by the condition: if $r_{pq} \rightarrow r_{uv}$ then $R_p \Rightarrow R_u$. A binary operation γ is defined over set R in such a way that $\gamma(\langle\pi_i, k_i\rangle, \langle\pi_u, k_u\rangle) = \langle\pi, k_i\rangle$, where

$$\pi = \begin{cases} (j_1, \ldots, j_{k_i}, j_{k_u+1}, \ldots, j_{\ell_u}), & \text{if } k_u \ne l_u, \\ (j_1, \ldots, j_{k_i}), & \text{otherwise.} \end{cases}$$

A set $\mathcal{P}$ of permutations is called *normalized* (with respect to the partitions $\{R_p\}$ and $\{r_{pq}\}$), if for any two elements $\langle\pi_i, k_i\rangle \in R$ and $\langle\pi_u, k_u\rangle \in R$ the conditions $R(\langle\pi_i, k_i\rangle) \Rightarrow R(\langle\pi_u, k_u\rangle)$ and $\gamma(\langle\pi_i, k_i\rangle, \langle\pi_u, k_u\rangle) = \langle\pi, k_i\rangle$ imply that

(i) $\pi \in \mathcal{P}$.
(ii) $r(\langle\pi, k_i\rangle) \rightarrow r(\langle\pi_i, k_i\rangle)$ and $r(\langle\pi, k_i - 1\rangle) \rightarrow r(\langle\pi_i, k_i - 1\rangle)$ for $k_i > 1$.
(iii) $r(\langle\pi, k_u + 1\rangle) \rightarrow r(\langle\pi_u, k_u + 1\rangle)$, provided that $k_u < l_u$.

A function $\Phi(\cdot, \cdot)$ in (5) is called *monotone-recursive* if it is non-decreasing with respect to each of its arguments, i.e., if $a < b$ then $\Phi(a, r) \le \Phi(b, r)$ for all $r \in \{r_{pq}\}$ and, additionally, if $r_1 \rightarrow r_2$ then $\Phi(c, r_1) \le \Phi(c, r_2)$ for all c.

In Levin and Tanaev (1970, 1978) the properties of the normalized sets of permutations are established. The authors investigate the method of determining so-called (s, t)-neighborhoods, which appears to be one of the most popular ways of forming the partitions $\{R_p\}$ and $\{r_{pq}\}$ for practical problems. For the kth element in a permutation $\pi = (j_1, j_2, \ldots, j_\ell)$, its *$(s, t)$-neighborhood* is defined as an ordered triple $Q_{st}(\pi, k) = \langle Q, \sigma_s, \sigma_t\rangle$, where $Q = \{j_1, j_2, \ldots, j_{k-s}\}$ is a set which is empty is $k < s$, while σ_s, σ_t are permutations such that $\sigma_s = (j_{\max\{1, k-s+1\}}, \ldots, j_{k-1}, j_k)$ and $\sigma_t = (j_{k+1}, j_{k+2}, \ldots, j_{\min\{\ell, k+t\}})$.

The following optimality criterion for the problem of minimizing function $g(\pi)$ has been established. For a set $R' \subseteq R$, define $F(R') = \min\{f(\langle\pi_i, k\rangle | \langle\pi_i, k\rangle \in R')\}$. Let $\overline{R}$ be the set of all $R_j \in \{R_p\}$ for which there exists $R_i \in \{R_p\}$ such that $R_i \Rightarrow R_j$ and $F(R_i) \le F(R_j)$. Then there exists a permutation $\pi_{i^*} \in \mathcal{P}$ such that $f(\langle\pi_{i^*}, \ell_{i^*}\rangle) = \min\{g(\pi) | \pi \in \mathcal{P}\}$, and additionally the relations $R(\langle\pi_{i^*}, k\rangle) \notin \overline{R}$ and $f(\langle\pi_{i^*}, k\rangle) = F(R(\langle\pi_{i^*}, k\rangle))$ hold for all $k = 1, \ldots, \ell_{i^*}$. This criterion leads to the following two-level scheme of a solution procedure. At the lower level, the value $F(R_p)$ is determined for a fixed R_p, while at the higher level, the value of $F(R_p)$ is minimized over $\{R_p\}$. For the higher level problem, the recurrent relations have been derived that allow us to use the techniques typical for the methods of DP and of sequential analysis of variants (Mikhalevich 1965a, 1965b).

The results of these studies have been further extended to optimization over so-called "weakly normalized" sets of permutations (Levin 1980).

4.3 Scheduling with symmetric objective functions

A function $F(x_1, x_2, \ldots, x_n)$ is called *symmetric* if it does not depend on the order of the arguments, i.e., for any permutation $(j_1, j_2, \ldots, j_n)$ the equality $F(x_1, x_2, \ldots, x_n) = F(x_{j_1}, x_{j_2}, \ldots, x_{j_n})$ holds.

In most scheduling problems, it is required to minimize a *regular* function of the completion times of jobs, i.e., $F(C_1, C_2, \ldots, C_n)$ that is non-decreasing in any of its arguments. Many objective functions used in scheduling are symmetric. Examples of regular symmetric functions that are popular in scheduling include the maximum completion time (the makespan) $C_{\max} = \max\{C_j | j \in N\}$, the total completion time $\sum_{j \in N} C_j$, the sum of squared

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completion times $\sum_{j\in N} C_j^2$, the maximum tardiness $T_{\max} = \max\{\max\{C_j - d, 0\} | j \in N\}$ and the total tardiness $\sum_{j\in N} T_j = \sum_{j\in N} \max\{C_j - d, 0\}$ with respect to a common due date d, and many more. The study initiated by V.S. Tanaev and continued together with A.A. Gladky in the 1990s demonstrates that for numerous scheduling problems polynomial-time algorithms known for minimizing a particular function can be extended to minimizing an arbitrary regular symmetric function.

A problem of minimizing a generalized symmetric function G_{sym} is studied in Tanaev (1992). Here $G_{sym} = G_{sym}(f_1, f_2, \dots, f_n)$ is a composite symmetric function and f_j is a penalty associated with job $j \in N$. Let X be a set of feasible vectors $(f_1, f_2, \dots, f_n)$. A *minimal* element of the set X is a vector $(f_1^0, \dots, f_n^0)$ for which there exists no vector $(f_1, \dots, f_n) \in X$ such that $f_{i_k} \le f_{j_k}^0$, $k = 1, \dots, n$, and at least one of these inequalities is strict. Here $(i_1, \dots, i_n)$ and $(j_1, \dots, j_n)$ are permutations such that $f_{i_1} \le \dots \le f_{i_n}$ and $f_{j_1}^0 \le \dots \le f_{j_n}^0$. If set X contains a *unique* minimal element (accurate up to a permutation of its components), then it delivers the *minimum to any regular symmetric* function on X. Therefore, if for some scheduling problem there is an algorithm, which detects this unique minimal element, then the algorithm is applicable to the problem with any regular symmetric objective function. Notice that the minimum of an increasing symmetric function is always achieved at a minimal element if it exists.

Set X which possesses a unique minimal element is called *minorant*. Thus, the fact that the problem of minimizing an increasing symmetric function on a minorant set is solvable in polynomial time implies polynomial solvability of the problem to minimize any other regular symmetric function on this set. On the other hand, the NP-hardness of minimizing a regular symmetric function on a minorant set implies the NP-hardness of minimizing any increasing symmetric function on this set (Tanaev 1992, 1993). Examples of symmetric and non-symmetric functions, as well as examples of problems in which a unique minimal element exists and does not exist can be found in Tanaev (1992).

For instance, Tanaev (1992) considers problem $1|p_i < p_j \Rightarrow w_i \ge w_j|\sum w_j U_j$ with agreeable processing times and weights of the jobs to minimize the weighted number of late jobs; here $U_j = 1$ if job j is late, otherwise $U_j = 0$. This problem is solvable in $O(n \log n)$ time as shown in Gordon and Tanaev (1971). Since the set of feasible values $(w_1U_1, \dots, w_nU_n)$ is minorant in this problem, and the function $\sum w_j U_j$ is symmetric and increasing on this set, the algorithm in Gordon and Tanaev (1971) is optimal for problem $1|p_i < p_j \Rightarrow w_i \ge w_j|G_{sym}(w_1U_1, \dots, w_nU_n)$ with any symmetric regular function $G_{sym}(w_1U_1, \dots, w_nU_n)$. Another example of this approach is problem $1\|F_{sym}(C_1, \dots, C_n)$. Since problem $1\|\sum C_j$ is solvable by the *Shortest Processing Time* (*SPT*) rule as established in Smith (1956), the set of feasible values $(C_1, \dots, C_n)$ is minorant in this problem, and $\sum C_j$ is an increasing symmetric function, it follows that the SPT rule determines an optimal solution for problem $1\|F_{sym}(C_1, \dots, C_n)$ with any regular symmetric objective function of job completion times. Examples are $F_{sym} = \max\{\max\{(C_j)^\alpha - d, 0\} | j \in N\}$ and $F_{sym} = \sum_j \max\{(C_j)^\alpha - d, 0\}$ for $\alpha > 0$.

The results from Tanaev (1992) are extended to the case of non-zero job release dates r_j with preemptions allowed in Tanaev (1993). In particular, problem $1|r_j, pmtn|F_{sym}(C_1, \dots, C_n)$ with an arbitrary regular symmetric function can be solved in $O(n \log n)$ time by the algorithm described in Baker (1974), developed for problem $1|r_j, pmtn|\sum C_j$ to minimize the sum of completion times.

A further extension of this approach to scheduling on identical parallel machines is done in Tanaev and Gladky (1994a, 1994b). Among the implications of the study in Tanaev and Gladky (1994a) is the fact that problem $P2|p_j = 1, prec|F_{sym}(C_1, \dots, C_n)$ of scheduling unit-time jobs on two parallel identical machines under arbitrary precedence constraints to minimize an arbitrary regular symmetric function can by solved in $O(n^2)$ time by the algorithm presented in Coffman and Graham (1972), originally developed for problem $P2|p_j = 1, prec|C_{\max}$ to minimize the makespan and applicable for minimizing $\sum C_j$. Similarly, Tanaev and Gladky (1994b) prove that the $O(n)$ time algorithm in Hu (1961), originally developed for problem $P|p_j = 1, out-tree|C_{\max}$ of scheduling unit-time jobs on m parallel identical machines under tree-like precedence constraints to minimize the makespan, solves problem $P|p_j = 1, out-tree|F_{sym}(C_1, \dots, C_n)$ to minimize an arbitrary regular symmetric function.

The problem of scheduling unit-time jobs on uniform parallel machines to minimize the makespan is studied in Kovalyov and Shafransky (1998). Some jobs may require a unit of an additional renewable resource during their execution, whose total amount is upper bounded at each time instant. The proposed polynomial time algorithm is shown to find the unique minimal element; thus, the algorithm can be used for minimizing any regular symmetric function over this set.

The open shop problem with unit-time jobs is considered in Shakhlevich (2005), the algorithms for finding schedules that minimize any regular symmetric convex function and any regular symmetric concave function are presented.

Scheduling problems in which human resources have to operate in a contaminated area are studied in Janiak and Kovalyov (2006), and for some problems of this range algorithms for minimizing an arbitrary regular symmetric objective function are presented.

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5 Scheduling with transfer operators

The work of V.S. Tanaev in early the 1960s on scheduling with transfer operators falls into several categories of modern scheduling theory, such as scheduling with transportation considerations or transportation/communication delays, robotic flow shop scheduling, and cyclic scheduling; see, e.g., Brucker et al. (2004), Dawande et al. (2005), Levner et al. (2007) for recent reviews of the relevant areas.

The models studied in Tanaev (1964a, 1964b), Blokh and Tanaev (1966), Tanaev and Shkurba (1975) are related to a periodic flow shop with m machines in which either a finite number or an infinite number of non-preemptive jobs that belong to one or several families have to be scheduled. Jobs of the same family are identical. They are transferred from the previous machine to the next machine down the route by an operator of unit capacity. There can be one, several or an unlimited number of such operators. Their movements from one machine to another can take a given time or can be instantaneous. The machine setup times between any two jobs either from the same family or from different families can be given. A schedule is characterized by the job start times on the machines and by the routes of the operators. The objective is to maximize the system's throughput, which is the average number of jobs completed per time unit.

In the 1960s, computational complexity analysis was not yet established and studies on the existence of problem solutions attracted attention of mathematicians. In his first publications on scheduling with transfer operators, Tanaev concentrated on this latter issue. For the case of an infinite number of jobs, he generalized the results in Suprunenko et al. (1962), Aizenshtat (1963), which were obtained for so-called *primitive* cyclic processes, and established the necessary and sufficient conditions for the existence of feasible solutions for the scheduling problems with transfer operators, and those for the existence of an optimal solution, which is *periodic*. He used a concept of *incompatible* time intervals for job transfer, which was later used for other scheduling problems under the name *forbidden intervals* by several authors, see Levner et al. (2007).

In Blokh and Tanaev (1966) a periodic schedule is proved to be asymptotically optimal for the problem with a single family and an infinite number of jobs. The authors further reduced finding an optimal periodic schedule to finding a circuit in a digraph with the maximum ratio of two sums, where one sum is associated with arc weights and the other sum with arc lengths. Similar results were obtained in the theory of discrete control in Romanovskii (1964, 1967). As pointed out in Levner et al. (2007), these results were independently rediscovered in different forms in the 1970s–2000s by many well-known mathematicians. Since the 1960s, the problem of finding maximum or minimum ratio circuit in a digraph has been very popular in combinatorial optimization. It has been studied, among others, in Dantzig et al. (1967), Megiddo (1978), Karp (1978); Karp and Orlin (1981), Young et al. (1991), Orlin and Ahuja (1992).

If the number of families and the number of jobs are both finite and the number of transfer operators is unlimited, the original problem is reduced in Tanaev (1964b) to a problem, which can now be classified as a *Period*(*ic*) *Traveling Salesman Problem* (*PTSP*) or *Period*(*ic*) *Vehicle Routing Problem* (*PVRP*). In the latter problems, each city must be visited a given number of times. Tanaev provides an Integer Linear Programming formulation and develops an interesting solution approach, which is to convert solutions of an assignment problem into the required solutions of PTSP by means of introducing additional linear constraints. Active studies of PTSP and PVRP began in the 1980s and still continue to expand. Related information can be found in Christofides and Beasley (1984), Laporte and Osman (1995).

Given the optimal job processing intervals in the case of unlimited number of operators, the problem of minimizing the number of required transfer operators is reduced in Blokh and Tanaev (1966) to the classical problem of decomposing a poset into the minimum number of chains, whose well-known properties were established in Dilworth (1950), and a solution algorithm was suggested in Ford and Fulkerson (1962). This type of reduction was later applied to the interval scheduling problems, in which there are no operators but the jobs should be processed in their given time intervals; see the surveys (Kolen et al. 2007; Kovalyov et al. 2007).

6 Parametric decomposition of optimization problems

Apart from scheduling, another important direction of research conducted by V.S. Tanaev is related to the development of decomposition solution techniques for complex optimization problems. In this section, we review major achievements in this area.

Decomposition methods in mathematical programming were originated in Dantzig and Wolfe (1960), Kornai and Liptak (1965) who suggested two approaches to decomposing linear programming problems: column generation and constraint separation. Later on, approaches based on constraint relaxation, constraint fixing, generation and relaxation of constraints, variables aggregation, the use of the Lagrangean function, and the use of a small parameter were developed.

Since the early 1970s, V.S. Tanaev together with G.M. Levin was developing a general theory of parametric decomposition of optimization problems (Levin and Tanaev 1974a, 1974b, 1977). Notice that the term "parametric decomposition" had been earlier coined in Ermoliev and Ermolieva (1972). The core of the theory is the idea of parame-

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trization of an initial problem A by introducing into it additional parameters and constraints in such a way that for fixed values of the parameters the obtained parametrized problem B would be substantially easier to solve than the initial problem. A special case of parametrization is fragmentary parameterization, which requires substitution of certain fragments of the objective function and those of constraints by parameters. In particular, it is desirable that subproblem B' obtained from B by fixing the introduced parameters would decompose into a collection of several simpler independent subproblems of a smaller dimension. A two-level solution scheme is used for solving the parameterized problem B: at the lower level, subproblem B' with fixed values of the parameters is solved, and at the upper level, a coordinating subproblem B'' is solved with a purpose of determining the optimal values of the parameters that define subproblem B'. A successive application of this technique leads to multilevel decomposition of the original problem.

A general scheme of parametric decomposition has been designed and sufficient conditions of its applicability have been established. Under these conditions, the links between the stationary domains and the local minima domains of the objective functions of the original problem A and those of the arising subproblems have been studied. Several reasons have been identified, due to which the stationary domains and the local minima domains of problem A generate similar domains in a lower level subproblem B' obtained as a result of decomposition. For a higher level coordinating subproblem B'', a classification of domains (stationary and local minima) has been obtained with respect to their relations to the analogous areas of the initial problem A. Several types of such domains have been identified, with only one type to be of essential impact on the complexity of solving subproblem B''. Each such domain (either stationary or local minima) in subproblem B'' is generated by the corresponding domain of problem A; thus, the number of such special local minima domains in B'' is no more than in A, so that subproblem B'' is no harder than the original problem.

The results of this study made the core of Tanaev's thesis for the Habilitated Doctor degree (1977), they are reported in the monographs (Levin and Tanaev 1978; Tanaev 1987) and reviewed in the survey (Verina et al. 1988).

Based on the previously obtained results, the enhanced parametric decomposition theory has emerged that uses parametrization as a common foundation for combined application of decomposition along with the embedding of the obtained subproblems into simpler and computationally easier problems. This extension of parametric decomposition theory was successfully completed in the 1990s by V.S. Tanaev together with G.M. Levin and L.F. Verina; see Verina et al. (1995), Levin and Tanaev (1998, 2002).

Using the schemes of parametric decomposition techniques, numerous decomposition methods for solving various problems of mathematical and discrete programming have been constructed by V.S. Tanaev and his colleagues (Verina 1985; Verina and Levin 1991; Guschinsky and Levin 1987, 1991; Guschinsky et al. 1991). These applications include also problems that arise in automated design, in particular in optimization of the structure and parameters of a multi-positional production system (Levin and Tanaev 1978; Dolgui et al. 2005, 2006a, 2006b; Guschinskaya et al. 2008), as well as in optimization of parameters of advanced multi-link transmissions (Levin et al. 2004; Guschinsky et al. 2006; Dolgui et al. 2007). Based on the obtained results, decision support systems for designing the mentioned objects have been developed (Dolgui et al. 2008a, 2008b). These systems have been implemented at major relevant production enterprises of Belarus: Minsk and Baranovichi transfer line plants, Minsk tractor plant, concern AMKODOR.

7 Books and surveys

In this section, we review the books and surveys written or co-authored by V.S. Tanaev on various aspects of scheduling.

One of the major roles of V.S. Tanaev was that of a promoter of scheduling research in the former Soviet Union. He has co-authored several research monographs, including the most influential (Tanaev and Shkurba 1975; Tanaev et al. 1984a, 1989a, 1989b), that were aimed at getting Russian-speaking researchers and students to become familiar with the most essential results in the area. A systematic and structural approach, so characteristic for a scientific style of V.S. Tanaev, makes these books useful even after many years after their first publication.

It is worth mentioning that before the 1990s contacts between the researchers from the West and from the Eastern Europe were limited. The papers published in the West became known to the East European colleagues after a considerable delay. Besides, in almost all areas of science, including Operational Research and Scheduling in particular, only a few libraries of the former Soviet Union, normally located at major cities and scientific centers (Moscow, Leningrad, Novosibirsk, Kiev), were able to buy foreign journals and books. It was not uncommon that less than five copies of a journal, even of a high international rank, were available in the whole country. Thus, a secondary goal of the mentioned monographs was to present the reader with the results published in hardly accessible sources.

The monograph (Tanaev and Shkurba 1975) was produced by the main Soviet publisher of scientific literature Nauka ("Science"), Moscow. Accidentally, the same year the same publisher issued the Russian translation of the famous scheduling book (Conway et al. 1967). Thus, 1975 was indeed a very important year for scheduling in the Soviet Union. Unlike the book (Conway et al. 1967), the mono-

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graph (Tanaev and Shkurba 1975) concentrates on deterministic scheduling models. It describes several useful techniques of combinatorial analysis earlier discussed in this paper, including the pairwise interchange method (see Sect. 2), modeling with mixed graphs (see Sect. 3) and optimization of recursive functions over a set of permutations (see Sect. 4.2). There are chapters on single machine models and on those with identical parallel machines; in particular the issues of the existence of non-preemptive schedules are addressed (see Sect. 4.1 above). Two chapters are devoted to the flow shop models. For the two-machine case, a generalization of the classical algorithm from Johnson (1954) is shown to work in the presence of various time lags, not necessarily positive. For the multi-machine case a branch-and-bound algorithm is detailed. The job shop chapter discusses a mixed graph representation and mathematical programming formulations of the relevant models. A separate chapter treats scheduling models with transfer operators, for both finite and infinite numbers of operators (see Sect. 5 above).

The two books (Tanaev et al. 1984a, 1989a, 1989b), also issued by Nauka, should be seen as a two-volume monograph aimed at covering the most essential scheduling results at the time; exactly in such a two-volume form the books were translated into English in 1994. The years elapsed since the publication of the previous book (Tanaev and Shkurba 1975) were probably most important for forming the modern shape of scheduling theory. That was the time of the arrival of the theory of computational complexity, the time of creating the famous three-field notation system for scheduling models and of other developments captured in a series of influential surveys with the core team of authors consisting of E.L. Lawler, J.K. Lenstra and A.H.G. Rinnooy Kan; see the seminal and most quoted survey (Graham et al. 1979). There was a need to reflect those changes and achievements in a systematic way, and in a manner suitable for a Russian-speaking reader.

The monograph (Tanaev et al. 1984a) gives a comprehensive presentation of the results known at the time regarding the single-stage scheduling systems, i.e., the systems with a single machine and those with parallel machines (identical, uniform and unrelated). Unlike many other books on scheduling, this monograph is not organized on the model-after-model principle. Apart from an introductory chapter of general combinatorial techniques, it contains a large chapter on polynomial-time algorithms, a chapter that presents the theory of minimization of priority-generating functions (see Sect. 2 of this paper) and a chapter on the NP-hard scheduling problems. In turn, the chapter on NP-hardness is split into sections from the point of view of the method of proof: one section shows how Partition problem can be used for polynomial reduction, then 3-Partition, vertex cover, clique, etc. Due to space restrictions, the book concentrates on the aspects of scheduling that are mainly of academic interest; the exact enumerative methods, as well as approximation and heuristic algorithms are not discussed in the main body of the book. For the English version, additions and corrections of the original text were performed, including an added appendix on approximation algorithms in single-stage scheduling.

The monograph (Tanaev et al. 1989b) treats the models of multi-stage processing, including the flow shop, the job shop and the open shop, and it is split into chapters according to these models. A separate chapter describes the use of mixed graphs and multigraphs for modeling and optimization of complex processing systems (see Sect. 3 of this paper). As in the previous book, the main stress is on the complexity issues, and for each considered model an attempt is made to provide a borderline between the versions that are polynomially solvable and those which are NP-hard. Approximation algorithms and their worst-case analysis (an insufficiently developed area at the time of publication) are briefly discussed. The mixed graph chapter presents branch-and-bound algorithms for the relevant problems. Heuristic and local search algorithms are not discussed in the main body of the book. Several updates and corrections are performed for the English edition.

All three books above have a common feature that V.S. Tanaev saw necessary for a scientific monograph: the body of each chapter is a smooth text split into sections and subsections, and does not contain any references to the original publications. Each chapter is accompanied by a section that provides bibliographic notes on its content.

What makes the books useful sources even now, is their extensive lists of references, in Russian and other languages, mainly English. For example, the book (Tanaev et al. 1989b) quotes 839 publications, and extra 111 references are added for the English version of 1994.

An important side of V.S. Tanaev's scientific activities is related to pedagogy. He lectured at the Belarussian State University, Minsk, took part in the final vivas for its undergraduates (where he never missed an opportunity to recruit an able student to join his research laboratory). He actively participated in various educational programmes in Belarus. V.S. Tanaev supervised numerous PhD students and understood that the earlier a young researcher gets exposed to the area of her or his future research, the higher the chances are for eventual success.

Among the publications of V.S. Tanaev there is a rather thin brochure (Tanaev 1988). "Scheduling theory", it is written to be understandable to an able high school student and is meant to attract young people to scheduling. And, who knows, may be some of the young readers will choose scheduling as their future career?

Although scheduling theory was not a standard compulsory course at a Soviet University, at several places this

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and similar topics were taught as options or courses of specialization for senior undergraduates. With that in mind, V.S. Tanaev co-authored a textbook (Sotskov et al. 1994). Written not long after the completion of the monograph (Tanaev et al. 1989b), the textbook was aimed at delivering major scheduling results in a friendly manner with additional exercises varying from numerical examples to proofs of different degrees of hardness. The book was recommended as a textbook for the students in Applied Mathematics.

The last monograph on scheduling co-authored by Tanaev is Tanaev et al. (1998). It is devoted to various group technology and batch scheduling models. To cope with the variety of the models, the unified terminology and notation system is developed. The problems are classified with regard to the type of the processing system. A fairly complete analysis of their computational complexity is provided, and selected efficient solution methods are described. A considerable part of the book demonstrates new applications of priority-generating functions (see Sect. 2 of this paper) to solving group technology scheduling problems under precedence constrains.

Apart from the reviewed books, V.S. Tanaev has co-authored several surveys on various aspects of scheduling. The survey (Kovalyov et al. 1989) addresses approximation scheduling algorithms, and at the time of publication was probably the most comprehensive review on the topic. A good overview of the scheduling contributions of the Minsk Group led by V.S. Tanaev is presented in Sotskov and Tanaev (1994). A detailed survey on the stability issues in scheduling (see Sect. 3) is contained in Sotskov et al. (1998a). In Gordon and Tanaev (2001) the reader will find a concise survey on the single machine scheduling problems with due dates and deadlines.

Theoretical findings of V.S. Tanaev and his collaborators resulted in software packages on timetabling (Barkan and Tanaev 1970), multi-step optimization of monotone-recursive functions (Tanaev et al. 1984b, 1986b, 1986c) and scheduling (Tanaev et al. 1986a, 1987, 1989a, 1989b).

Edited by V.S. Tanaev, several books of collected articles on algorithms and software for optimization problems were published in 1970–90s years at the Institute of Engineering Cybernetics of the Academy of Sciences of BSSR (Tanaev 1977b, 1979b, 1980, 1981, 1982, 1983, 1984, 1985, 1989, 1990, 1991).

As far other research interests of V.S. Tanaev are concerned, the books (Levin and Tanaev 1978; Tanaev 1987) and the survey (Verina et al. 1988) on parametric decomposition of optimization problems have been discussed in Sect. 6.

The book (Tanaev and Povarich 1974) addresses the issues of application of so-called tables of usage to decision-making in automated design. The properties of these tables are studied, the methods of synthesis of the graphs-schemes of selecting decisions by these tables are developed and implemented. The software package is described that allows generating computer programs of decision-makings based on the graph-schemes.

8 Conclusion

The purpose of this paper is to review several areas of research conducted by V.S. Tanaev and to demonstrate an impact that he had on the development of scheduling theory and optimization, especially in the countries of the former Soviet Union. The authors are proud to be the members of the Minsk Group created and led by V.S. Tanaev till his last days, and for each of us an opportunity to work with him has been one of the most important factors of shaping one as a researcher.

References

Aizenshtat, V. S. (1963). Multi-operator cyclic processes. *Doklady Akademii Nauk BSSR*, *7*(4), 224–227 (in Russian).

Al-Anzi, F., Sotskov, Yu. N., Allahverdi, A., & Andreev, G. (2006). Using mixed graph coloring to minimize total completion time in job shop scheduling. *Mathematics of Computation*, *182*, 1137–1148.

Alyushkevich, V. B., & Sotskov, Yu. N. (1989). Stability in the problems of production planning. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *3*, 102–107 (in Russian).

Baker, K. R. (1974). *Introduction to sequencing and scheduling*. New York: Wiley.

Baker, K. R., Lawler, E. L., Lenstra, J. K., & Rinnooy Kan, A. H. G. (1983). Preemptive scheduling of a single machine to minimize maximum cost subject to release dates and precedence constraints. *Operations Research*, *31*(2), 381–386.

Barkan, S. A., & Tanaev, V. S. (1970). On constructing class schedules. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *1*, 76–81 (in Russian).

Blazewicz, J. (1976). Scheduling dependent tasks with different arrival times to meet deadlines. In E. Gelenbe & H. Beilner (Eds.), *Modelling and Performance Evaluation of Computer Systems* (pp. 57–65). Amsterdam: North Holland.

Blazewicz, J., Ecker, K. H., Pesch, E., Schmidt, G., & Weglarz, J. (2007). *Handbook on scheduling*. Berlin: Springer.

Blokh, A. S., & Tanaev, V. S. (1966). Multioperator processes. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *2*, 5–11 (in Russian).

Brucker, P., Knust, S., Cheng, T. C. E., & Shakhlevich, N. V. (2004). Complexity results for flow-shop and open-shop scheduling problems with transportation delays. *Annals of Operations Research*, *129*(1–4), 81–106.

Christofides, N., & Beasley, J. E. (1984). Period routing problem. *Networks*, *14*(2), 237–256.

Coffman, E. G., & Graham, R. L. (1972). Optimal scheduling for two-processor systems. *Acta Informatica*, *1*(3), 200–213.

Conway, R. W., Maxwell, W. L., & Miller, L. W. (1967). *Theory of scheduling*. Reading: Addison-Wesley.

Dantzig, G. B., & Wolfe, P. (1960). Decomposition principle for linear programs. *Operations Research*, *8*(1), 101–112.

Dantzig, G. B., Blattner, W., & Rao, M. R. (1967). Finding a cycle in a graph with minimum cost to time ratio with application to a ship routing problem. In P. Rosenstiehl (Ed.), *Theory of graphs* (pp. 77–84). Paris/New York: Dunod/Gordon & Breach.

Dawande, M., Geismer, H. N., Sethi, S. P., & Sriskandarajah, C. (2005). Sequencing and scheduling in robotic cells: Recent developments. *Journal of Scheduling*, *8*(5), 387–426.

Dilworth, R. P. (1950). A decomposition for partially ordered sets. *Annals of Mathematics*, *51*, 161–166.

Dolgui, A., Levin, G., & Louly, M. A. (2005). Decomposition approach for a problem of lot-sizing and sequencing under uncertainties. *International Journal of Computer Integrated Manufacturing*, *18*(5), 376–385.

Dolgui, A., Finel, B., Guschinskaya, O., Guschinsky, N., Levin, G., & Vernadat, F. (2006a). Balancing large-scale machining lines with multi-spindle heads using decomposition. *International Journal of Production Research*, *44*(18–19), 4105–4120.

Dolgui, A., Guschinsky, N., & Levin, G. (2006b). A decomposition method for transfer line life cycle cost optimization. *Journal of Mathematical Modeling and Algorithms*, *5*, 215–238.

Dolgui, A., Guschinsky, N., & Levin, G. (2007). Optimisation of power transmission systems using a multilevel decomposition approach. *RAIRO–Operations Research*, *41*, 213–229.

Dolgui, A., Guschinskaya, O., Guschinsky, N., & Levin, G. (2008a). Decision making and support tools for design of machining systems. In F. Adam & P. Humphreys (Eds.), *Encyclopedia of decision making and decision support technologies* (pp. 155–164). Hershey: Idea Group Inc.

Dolgui, A., Guschinsky, N., & Levin, G. (2008b). Decision making and support tools for design of transmission systems. In F. Adam & P. Humphreys (Eds.), *Encyclopedia of decision making and decision support technologies* (pp. 165–175). Hershey: Idea Group Inc.

Emelichev, V. A., Girlich, E. N., Nikulin, Y. V., & Podkopaev, D. P. (2002). Stability and regularization radius of vector problems of integer linear programming. *Optimization*, *51*(4), 645–676.

Ermoliev, Y. G., & Ermolieva, L. G. (1972). Method of parametric decomposition. *Kibernetika*, *1*, 66–69.

Ford, L. R. Jr., & Fulkerson, D. R. (1962). *Flows in networks*. Princeton: Princeton University Press.

Gladky, A. A., Shafransky, Y. M., & Strusevich, V. A. (2004). Flow shop scheduling problems under machine-dependent precedence constraints. *Journal of Combinatorial Optimization*, *8*, 13–28.

Gordon, V. S., & Shafransky, Y. M. (1977). On a class of scheduling problems with partially ordered jobs. In *Proceedings of the 4-th all-union conference on theoretical cybernetics problems*, Novosibirsk, August 30–September 1 (pp. 101–103) (in Russian).

Gordon, V. S., & Shafransky, Y. M. (1978a). Optimal ordering with series-parallel precedence constraints. *Doklady Akademii Nauk BSSR*, *22*(3), 244–247 (in Russian).

Gordon, V. S., & Shafransky, Y. M. (1978b). The decomposition approach to minimizing functions over a set of permutations of partially ordered elements. In *Proceedings of the 5-th all-union conference of complex system control* (pp. 51–56). Alma-Ata (in Russian).

Gordon, V. S., & Shafransky, Y. M. (1978c). On optimal ordering with series-parallel precedence constraints. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *5*, 135 (in Russian).

Gordon, V. S., & Strusevich, V. A. (1999). Earliness penalties on a single machine subject to precedence constraints: SLK due date assignment. *Computers & Operations Research*, *26*, 157–177.

Gordon, V. S., & Tanaev, V. S. (1971). Single-machine deterministic scheduling with step functions of penalties. In *Computers in engineering* (pp. 3–8). Minsk (in Russian).

Gordon, V. S., & Tanaev, V. S. (1973a). Single-machine deterministic scheduling with tree-like ordered jobs and exponential penalty functions. In *Computers in engineering* (pp. 3–10). Minsk (in Russian).

Gordon, V. S., & Tanaev, V. S. (1973b). Preemptions in deterministic systems with parallel machines and different release dates of jobs. In *Optimization of systems of collecting, transfer and processing of analogous and discrete data in local information computing systems. Materials of the 1st joint Soviet-Bulgarian seminar. Institute of Engineering Cybernetics of Academy of Sciences of BSSR—Institute of Engineering Cybernetics of Bulgarian Academy of Sciences* (pp. 36–50). Minsk (in Russian).

Gordon, V. S., & Tanaev, V. S. (1973c). Due dates in single-stage deterministic scheduling. In *Optimization of systems of collecting, transfer and processing of analogous and discrete data in local information computing systems. Materials of the 1st joint Soviet-Bulgarian seminar, Institute of Engineering Cybernetics of Academy of Sciences of BSSR—Institute of Engineering Cybernetics of Bulgarian Academy of Sciences* (pp. 54–58). Minsk (in Russian).

Gordon, V. S., & Tanaev, V. S. (1983). On minmax problems of scheduling theory for a single machine. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *3*, 3–9 (in Russian).

Gordon, V. S., & Tanaev, V. S. (2001). Scheduling decisions for the systems with deadlines. In Z. Binder (Ed.), *Proceedings of the 2nd IFAC/IFIP/IEEE conference, management and control of production and logistics* (vol. 2, pp. 687–690). Elmsford: Pergamon.

Gordon, V. S., Proth, J.-M., & Strusevich, V. A. (2005). Single machine scheduling and due date assignment under series-parallel precedence constraints. *Central European Journal of Operations Research*, *13*, 15–35.

Gordon, V. S., Potts, C. N., Strusevich, V. A., & Whitehead, J. D. (2008). Single machine scheduling models with deterioration and learning: Handling precedence constraints via priority generation. *Journal of Scheduling*, *11*, 357–370.

Graham, R. L., Lawler, E. L., Lenstra, J. K., & Rinnooy Kan, A. H. G. (1979). Optimization and approximation in deterministic scheduling: a survey. *Annals of Discrete Mathematics*, *5*, 287–326.

Guschinskaya, O., Dolgui, A., Guschinsky, N., & Levin, G. (2008). A heuristic multi-start decomposition approach for optimal design of serial machining lines. *European Journal of Operational Research*, *189*(3), 902–913.

Guschinsky, N. N., & Levin, G. M. (1987). Two-level optimization of a composite function and its application to a problem of path optimization in a graph. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *3*, 3–9 (in Russian).

Guschinsky, N. N., & Levin, G. M. (1991). Minimization of a monotone superposition of recurrent-monotone functions over the set of parametrized paths in a digraph. *Sistemy Modelirovaniya*, *17*, 167–178 (in Russian).

Guschinsky, N. N., Levin, G. M., & Tanaev, V. S. (1991). Parametric decomposition of problems of minimizing composite functions on parametrized paths in a digraph. *Soviet Journal of Computer and Systems Sciences*, *29*(6), 31–42 (translated from Russian, Izvestiya AN SSSR. Seria Tekhnicheskaya Kibernetika, 1990).

Guschinsky, N. N., Levin, G. M., & Dolgui, A. B. (2006). *Decision support for design of power transmissions*. Minsk: Belaruskaya Navuka (in Russian).

Hansen, P., Kuplinsky, J., & de Werra, D. (1997). Mixed graph colorings. *Mathematical Methods of Operational Research*, *45*, 145–160.

Hardy, G. H., Littlewood, J. E., & Polya, G. (1934). *Inequalities*. London: Cambridge University Press.

Horn, W. A. (1972). Single-machine job sequencing with treelike precedence ordering and linear delay penalties. *SIAM Journal of Applied Mathematics*, *23*, 189–202.

Horn, W. A. (1974). Some simple scheduling algorithms. *Naval Research Logistics Quarterly*, *21*(1), 177–185.

Hu, T. C. (1961). Parallel sequencing and assembly line problems. *Operations Research*, *9*, 841–848.

Jackson, J. R. (1955). *Scheduling a production line to minimize maximum tardiness* (Research Report 43, Management Science Research Project). University of California, Los Angeles, USA.

Janiak, A., & Kovalyov, M. Y. (2006). Scheduling in a contaminated area: a model and polynomial algorithms. *European Journal of Operational Research*, *173*, 125–132.

Janiak, A., Shafransky, Y. M., & Tuzikov, A. (2001). Sequencing with ordered criteria, precedence and group technology constraints. *Informatica*, *12*(1), 61–88.

Johnson, S. M. (1954). Optimal two- and three-stage production schedules with setup times included. *Naval Research Logistics Quarterly*, *1*, 61–68.

Karp, R. M. (1978). A characterization of the minimum cycle mean in a digraph. *Discrete Mathematics*, *23*, 309–311.

Karp, R. M., & Orlin, J. B. (1981). Parametric shortest path algorithms with an application to cyclic staffing. *Discrete Applied Mathematics*, *3*(1), 37–45.

Kladov, G. K., & Livshitz, E. M. (1968). On a scheduling problem to minimize the total penalty. *Kibernetika*, *6*, 99–100 (in Russian).

Kolen, A. W. J., Lenstra, J. K., Papadimitriou, C. H., & Spieksma, F. C. R. (2007). Interval scheduling: a survey. *Naval Research Logistics*, *54*, 530–543.

Kornai, J., & Liptak, T. (1965). Two-level planning. *Econometrica*, *33*, 141–169.

Kovalyov, M. Y., & Shafransky, Y. M. (1998). Uniform machine scheduling of unit-time jobs subject to resource constraints. *Discrete Applied Mathematics*, *84*, 253–257.

Kovalyov, M. Y., & Tuzikov, A. V. (1994). Sequencing groups of jobs on a single machine subject to precedence constraints. *Applied Mathematics and Computer Science*, *4*(4), 635–641.

Kovalyov, M. Y., Shafransky, Y. M., Strusevich, V. A., Tanaev, V. S., & Tuzikov, A. V. (1989). Approximation scheduling algorithms: a survey. *Optimization*, *20*(6), 859–878.

Kovalyov, M. Y., Ng, C. T., & Cheng, T. C. E. (2007). Fixed interval scheduling: models, applications, computational complexity and algorithms. *European Journal of Operational Research*, *178*, 331–342.

Kruger, K., Sotskov, Yu. N., & Werner, F. (1998). Heuristic for generalized shop scheduling problems based on decomposition. *International Journal of Production Research*, *36*(11), 3013–3033.

Lambin, N. V., & Tanaev, V. S. (1970). On circuit-free orientation of mixed graphs. *Doklady Akademii Nauk BSSR*, *14*(9), 780–781 (in Russian).

Laporte, G., & Osman, I. H. (1995). Routing problems: a bibliography. *Annals of Operation Research*, *61*(1), 227–262.

Leont'ev, V. K. (1975). Stability of the traveling salesman problem. *Zhurnal Vychislitel'noj Matematiki i Matematicheskoj Fiziki*, *15*(4), 1298–1309 (in Russian).

Levin, G. M. (1980). Towards optimization of functions recursively defined over weakly normalized sets of permutations. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *5*, 9–14.

Levin, G. M., & Tanaev, V. S. (1968). On a class of problems of combinatorial optimization. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *5*, 30–35 (in Russian).

Levin, G. M., & Tanaev, V. S. (1970). On the theory of optimization over a set of permutations. *Doklady Akademii Nauk BSSR*, *14*(7), 588–590 (in Russian).

Levin, G. M., & Tanaev, V. S. (1974a). Parametric decomposition of extremal problems. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *4*, 24–29 (in Russian).

Levin, G. M., & Tanaev, V. S. (1974b). Towards the theory of parametric decomposition of extremal problems. *Doklady Akademii Nauk BSSR*, *18*(10), 883–885 (in Russian).

Levin, G. M., & Tanaev, V. S. (1977). On parametric decomposition of extremal problems. *Kibernetika*, *3*, 123–128 (in Russian).

Levin, G. M., & Tanaev, V. S. (1978). *Decomposition methods in optimization of design decisions*. Minsk: Nauka i Tekhnika (in Russian).

Levin, G. M., & Tanaev, V. S. (1998). Parametric decomposition of optimization problems. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *4*, 121–131 (in Russian).

Levin, G. M., & Tanaev, V. S. (2002). Extended parametric decomposition of optimization problems: some properties and applications. *Iskusstvenny Intellekt*, *2*, 4–10 (in Russian).

Levin, G. M., Guschinsky, N. N., & Burdo, E. I. (2004). Optimization of transmission parameters of a cascade-reproduction structure. *Vestsi NAN of Belarus. Seryya Fizika-Matematychnykh Navuk*, *2*, 114–120.

Levner, E., Kats, V., & De Pablo, D. A. L. (2007). Cyclic scheduling in robotic cells: an extension of basic models in machine scheduling theory. In E. Levner (Ed.), *Multiprocessor scheduling: theory and applications* Vienna: Itech Education and Publishing.

Matyushkov, L. P., & Tanaev, V. S. (1967). A program generator for feasible schedules, I. In *Computers in engineering* (pp. 35–48). Minsk (in Russian).

Matyushkov, L. P., & Tanaev, V. S. (1968). A program generator for feasible schedules, II. In *Computers in engineering* (pp. 12–28). Minsk (in Russian).

McNaughton, R. (1959). Scheduling with deadlines and loss functions. *Management Science*, *6*(1), 1–12.

Megiddo, N. (1978). Combinatorial optimization with rational objective functions. In *Proceedings of the 10th annual ACM symposium on theory of computing* (pp. 1–12). San Diego.

Mikhalevich, V. S. (1965a). Sequential algorithms of optimization and their application: I. *Kibernetika*, *1*, 45–66.

Mikhalevich, V. S. (1965b). Sequential algorithms of optimization and their application: II. *Kibernetika*, *2*, 85–89 (in Russian).

Monma, C. L., & Sidney, J. B. (1979). Sequencing with series-parallel precedence constraints. *Mathematics of Operations Research*, *4*, 215–234.

Orlin, J. B., & Ahuja, R. K. (1992). New scaling algorithms for the assignment and minimum mean cycle problems. *Mathematical Programming*, *54*(1–3), 41–56.

Potts, C. N., & Strusevich, V. A. (2009). Fifty years of scheduling: a survey of milestones. *The Journal of the Operational Research Society*, *60*, S41–S68.

Ries, B. (2007). Coloring some classes of mixed graphs. *Discrete Applied Mathematics*, *155*, 1–6.

Ries, B., & de Werra, D. (2008). On two coloring problems in mixed graphs. *European Journal of Combinatorics*, *29*, 712–725.

Romanovskii, I. V. (1964). Asymptotic recursive relations of dynamic programming and optimal stationary control. *Doklady Akademii Nauk SSSR*, *157*(6), 1303–1306 (in Russian).

Romanovskii, I. V. (1967). Optimization of stationary control of a discrete deterministic process. *Kibernetika*, *3*, 66–78 (in Russian).

Rothkopf, M. H. (1966). Scheduling independent tasks on parallel processors. *Management Science*, *12*, 437–447.

Roy, B., & Sussmann, B. (1964). *Les problèmes d'ordonnancement avec contraintes disjonctives* (Note DS No 9 bis.). SEMA, Montrouge

Shafransky, Y. M. (1978a). Optimization for deterministic scheduling systems with tree-like partial order. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *2*, 119 (in Russian).

Shafransky, Y. M. (1978b). On optimal sequencing for deterministic systems with tree-like partial order. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *2*, 120 (in Russian).

Shafransky, Y. M., & Strusevich, V. A. (1998). The open shop scheduling problem with a given sequence on one machine. *Naval Research Logistics*, *45*, 705–731.

« JOSH 10951 layout: Large v.1.3.2 file: josh230.tex (DL) class: spr-twocol-v1.2 v.2011/02/19 Prn:2011/03/03; 8:31 p. 15/16»
« doctopic: OriginalPaper numbering style: ContentOnly reference style: apa»

Shakhlevich, N. V. (2005). Open shop unit-time scheduling problems with symmetric objective functions. *4OR*, *3*, 117–131.

Shakhlevich, N. V., Sotskov, Yu. N., & Werner, F. (1996). Adaptive scheduling algorithm based on the mixed graph model. *IEE Proceedings. Control Theory and Applications*, *43*(1), 9–16.

Smith, W. E. (1956). Various optimizers for single stage production. *Naval Research Logistics Quarterly*, *3*, 59–66.

Sotskov, Yu. N. (1991). Stability of an optimal schedule. *European Journal of Operational Research*, *55*, 91–102.

Sotskov, Yu. N. (1997). Mixed multigraph approach to scheduling jobs on machines of different types. *Optimization*, *42*, 245–280.

Sotskov, Yu. N., & Alyushkevich, V. B. (1988). Stability of optimal orientation of the edges of a mixed graph. *Doklady Akademii Nauk BSSR*, *32*(4), 108–111 (in Russian).

Sotskov, Yu. N., & Shakhlevich, N. V. (1995). NP-hardness of shop-scheduling problems with three jobs. *Discrete Applied Mathematics*, *59*, 237–266.

Sotskov, Yu. N., & Tanaev, V. S. (1974). On enumeration of the circuit-free digraphs generated by a mixed graph. *Vestsi Akademii Navuk BSSR, Seryya Fizika-Matematychnykh Navuk*, *2*, 16–21 (in Russian).

Sotskov, Yu. N., & Tanaev, V. S. (1976a). On one approach to enumeration of the circuit-free digraphs generated by a mixed graph. *Vestsi Akademii Navuk BSSR, Seryya Fizika-Matematychnykh Navuk*, *5*, 99–102 (in Russian).

Sotskov, Yu. N., & Tanaev, V. S. (1976b). A chromatic polynomial of a mixed graph. *Vestsi Akademii Navuk BSSR, Seryya Fizika-Matematychnykh Navuk*, *6*, 20–23 (in Russian).

Sotskov, Yu. N., & Tanaev, V. S. (1989). Construction of a schedule admissible for a mixed multi-graph. *Vestsi Akademii Navuk BSSR, Seryya Fizika-Matematychnykh Navuk*, *4*, 94–98 (in Russian).

Sotskov, Yu. N., & Tanaev, V. S. (1994). Scheduling theory and practice: Minsk group results. *Intelligent Systems Engineering*, *1994*, 1–8.

Sotskov, Yu. N., Strusevich, V. A., & Tanaev, V. S. (1994). *Mathematical models and methods of production planning*. Minsk: Universitetskoe (in Russian).

Sotskov, Yu. N., Leontev, V. K., & Gordeev, E. N. (1995). Some concepts of stability analysis in combinatorial optimization. *Discrete Applied Mathematics*, *58*, 169–190.

Sotskov, Yu. N., Sotskova, N. Yu., & Werner, F. (1997). Stability of an optimal schedule in a job shop. *Omega*, *25*(4), 397–414.

Sotskov, Yu. N., Tanaev, V. S., & Werner, F. (1998a). Stability radius of an optimal schedule: a survey and recent development. In *Industrial applications of combinatorial optimization* (pp. 72–108). Boston: Kluwer Academic.

Sotskov, Yu. N., Wagelmans, A. P. M., & Werner, F. (1998b). On the calculation of the stability radius of an optimal or an approximate schedule. *Annals of Operation Research*, *83*, 213–252.

Sotskov, Yu. N., Dolgui, A., & Werner, F. (2001). Mixed graph coloring for unit-time job-shop scheduling. *International Journal of Mathematical Algorithms*, *2*, 289–323.

Sotskov, Yu. N., Tanaev, V. S., & Werner, F. (2002). Scheduling problems and mixed graph colorings. *Optimization*, *51*(3), 597–624.

Strusevich, V. A. (1997a). Multi-stage scheduling problems with precedence constraints. In C. Mitchell (Ed.), *Applications of combinatorial mathematics* (pp. 217–232). London: Oxford University Press.

Strusevich, V. A. (1997b). Shop scheduling problems under precedence constraints. *Annals of Operation Research*, *69*, 351–377.

Suprunenko, D. A., Aizenshtat, V. S., & Metel'sky, A. S. (1962). A multistage technological process. *Doklady Akademii Nauk BSSR*, *6*(9), 541–544 (in Russian).

Tanaev, V. S. (1964a). On a flow shop scheduling problem with one operator. *Inzhenerno-Fizicheskij Zhurnal*, *3*, 111–114 (in Russian).

Tanaev, V. S. (1964b). On a scheduling problem. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *4*, 128–131 (in Russian).

Tanaev, V. S. (1964c). On scheduling theory. *Doklady Akademii Nauk BSSR*, *8*(12), 792–794 (in Russian).

Tanaev, V. S. (1965). Some objective functions of a single stage production. *Doklady Akademii Nauk BSSR*, *9*(1), 11–14 (in Russian).

Tanaev, V. S. (1967). On the number of permutations of *n* partially ordered elements. *Doklady Akademii Nauk BSSR*, *9*(3), 208 (in Russian).

Tanaev, V. S. (1968). A method to solve a discrete programming problem. *Ekonomika i Matematicheskie Metody*, *4*(5), 776–782 (in Russian).

Tanaev, V. S. (1973). Preemptions in deterministic scheduling systems with parallel identical machines. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *6*, 44–48 (in Russian).

Tanaev, V. S. (1975). Mixed (disjunctive) graphs in scheduling problems. In *Large systems of information and control* (p. 181). Sofia (in Russian).

Tanaev, V. S. (1977a). On optimization of recursive functions on a set of permutations. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *3*, 27–30 (in Russian).

Tanaev, V. S. (Ed.) (1977b). *Program library for solving extremal problems. Issue 1* (p. 1997). Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (1979a). On optimal partitioning a finite set into subsets. *Doklady Akademii Nauk BSSR*, *23*(1), 26–28 (in Russian).

Tanaev, V. S. (Ed.) (1979b). *Program library for solving extremal problems. Issue 2*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (Ed.) (1980). *Algorithms and programs for solving optimization problems*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (Ed.) (1981). *Methods and programs for solving extremal problems*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (Ed.) (1982). *Methods and programs for solving extremal problems and related issues*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (Ed.) (1983). *Algorithms and programs for solving optimization problems*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (Ed.) (1984). *Complexity and methods for solving optimization problems*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (Ed.) (1985). *Methods, algorithms and programs for solving extremal problems*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (1987). *Decomposition and aggregation in mathematical programming problems*. Minsk: Nauka i Technika (in Russian).

Tanaev, V. S. (1988). *Scheduling theory*. Moscow: Znanie (in Russian).

Tanaev, V. S. (Ed.) (1989). *Methods for solving extremal problems*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (Ed.) (1990). *Methods for solving extremal problems and related issues*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (Ed.) (1991). *Extremal problems of optimal planning and design*. Minsk: Institute of Engineering Cybernetics (in Russian).

Tanaev, V. S. (1992). Symmetric functions in scheduling theory (single machine problems with the same job release dates). *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *5–6*, 97–101 (in Russian).

Tanaev, V. S. (1993). Symmetric functions in scheduling theory (single machine problems with distinct job release dates). *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *1*, 84–87 (in Russian).

Journal ID: 10951, Article ID: 230, Date: 2011-03-03, Proof No: 1, UNCORRECTED PROOF

Tanaev, V. S., & Gladky, A. A. (1994a). Symmetric functions in scheduling theory (identical machines problems with ordered set of jobs). *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *4*, 81–84 (in Russian).

Tanaev, V. S., & Gladky, A. A. (1994b). *Symmetric functions in scheduling theory (parallel machines systems)* (Preprint 23). Minsk: Institute of Engineering Cybernetics.

Tanaev, V. S., & Gordon, V. S. (1983). On scheduling to minimize the weighted number of late jobs. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *6*, 3–9 (in Russian).

Tanaev, V. S., & Levin, G. M. (1967). On optimal behavior of limited memory systems. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *3*, 82–88 (in Russian).

Tanaev, V. S., & Povarich, M. P. (1974). *Synthesis of graph-schemes of decision-making*. Minsk: Nauka i Technika.

Tanaev, V. S., & Shkurba, V. V. (1975). *Introduction to scheduling theory*. Moscow: Nauka (in Russian).

Tanaev, V. S., Gordon, V. S., & Shafransky, Y. M. (1984a). *Scheduling theory. Single-stage systems*. Moscow: Nauka (in Russian); translated into English by Kluwer Academic Publishers, Dordrecht (1994).

Tanaev, V. S., Levin, G. M., & Sannikova, A. K. (1984b). *Program package for multi-step optimization (PP MODA)*. Minsk: Institute of Engineering Cybernetics.

Tanaev, V. S., Gordon, V. S., Sotskov, Yu. N., Yanova, O. V., Shafransky, Y. M., Gorokh, O. V., & Baranovskaya, S. M. (1986a). *A package of applied programs for solving sequencing problems (PAP RUPOR)*. Minsk: Institute of Engineering (in Russian).

Tanaev, V. S., Levin, G. M., Rozin, B. M., & Sannikova, A. K. (1986b). *A dialog system for synthesis of programs of multi-step optimization (MODA-7920)*. Minsk: Institute of Engineering (in Russian).

Tanaev, V. S., Levin, G. M., Rozin, B. M., & Sannikova, A. K. (1986c). A dialog system for design of programs of multi-step optimization MODA-7906. *Upravlyayushchie Sistemy i Machiny*, *3*, 95–99.

Tanaev, V. S., Gordon, V. S., Sotskov, Yu. N., Yanova, O. V., Shafransky, Y. M., Gorokh, O. V., & Baranovskaya, S. M. (1987). *A package of applied programs for solving sequencing problems (PAP RUPOR). Programs description*. Minsk: Institute of Engineering (in Russian).

Tanaev, V. S., Gordon, V. S., Sotskov, Yu. N., & Yanova, O. V. (1989a). A program package for solving scheduling theory problems. *Upravlyayuschie Sistemy i Machiny*, *4*, 107–111 (in Russian).

Tanaev, V. S., Sotskov, Yu. N., & Strusevich, V. A. (1989b). *Scheduling theory. Multi-stage systems*. Moscow: Nauka (in Russian); translated into English by Kluwer Academic Publishers, Dordrecht (1994).

Tanaev, V. S., Kovalyov, M. Y., & Shafransky, Y. M. (1998). *Scheduling theory. Group technologies*. Minsk: Institute of Engineering (in Russian).

Tuzikov, A. V., & Shafransky, Y. M. (1983). On problems of lexicographic minimization on a set of permutations. *Vestsi Akademii Navuk BSSR, Seryya Fizika-Matematychnykh Navuk*, *6*, 115 (in Russian).

Verina, L. F. (1985). Solution of some non-convex problems by reduction to convex mathematical programming. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *1*, 13–18 (in Russian).

Verina, L. F., & Levin, G. M. (1991). On a problem of optimization of a transfer function of network elements and its application to transmission design. *Vestsi Akademii Navuk BSSR. Seryya Fizika-Matematychnykh Navuk*, *2*, 28–32 (in Russian).

Verina, L. F., Levin, G. M., & Tanaev, V. S. (1988). Parametric decomposition of extremal problems—a general approach and some applications. *Soviet Journal of Computer and Systems Sciences*, *26*(4), 137–148 (translated from Russian, *Izvestiya AN SSSR. Seria Tekhnicheskaya Kibernetika*).

Verina, L. F., Levin, G. M., & Tanaev, V. S. (1995). Towards the theory of parametric decomposition and immersion of extremum problems. *Doklady Akademii Nauk BSSR*, *39*(4), 9–12 (in Russian).

Young, N. E., Tarjan, R. E., & Orlin, J. B. (1991). Faster parametric shortest path and minimum-balance algorithms. *Networks*, *21*(2), 205–221.

Zinder, Y. A. (1976). The priority solvability of a class of scheduling problems. In *Problems of design of automated systems of production control* (pp. 56–63). Kiev (in Russian).

Journal ID: 10951, Article ID: 230, Date: 2011-03-03, Proof No: 1, UNCORRECTED PROOF